Seasonally Varying Preferences: Theoretical Foundations for an Empirical Regularity*

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Seasonally Varying Preferences: Theoretical Foundations for an Empirical Regularity

Abstract

Equity and Treasury returns exhibit distinct seasonal cycles that are difficult to reconcile in a standard asset pricing framework. We investigate a representative agent asset pricing model in which we allow agents' preferences to cycle between two semi-annual seasons, with high risk aversion in one season and low risk aversion in the other. We explore whether any reasonable parameterization of this model can generate the observed seasonal patterns that equity and Treasury returns exhibit, and whether such a parameterization can match the observed magnitudes of seasonal return cycles. Calibrating to consumption data and incorporating the use of levered equity, we produce returns that match the qualitative and quantitative characteristics of observed equity and Treasury returns across the seasons. Specifically, risky asset returns are higher during the season when risk-free returns are lower, and vice versa; and further, risky asset returns vary seasonally more than risk-free returns. While a model with seasonally varying risk aversion is sufficient to match the directions of seasonal changes and rough magnitudes in returns, a novel result of our study is that additionally allowing seasonal variation in the intertemporal elasticity of substitution provides an even closer match to features of the data.

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A small but growing literature has emerged suggesting that the influence of seasonal variation in investor mood may lead to systematic seasonal effects in financial markets. Closely related to this study, Kamstra, Kramer, and Levi (2003) find that holding-period stock returns are higher during the 6-month period when many individual experience seasonal depression (the fall and winter seasons),[1] varying conditionally across the seasons as much as 12% (annualized), even after controlling for well-known seasonalities such as the January effect. Kamstra, Kramer, and Levi (2011) show that average returns to U.S. Treasury notes and bonds vary counter-cyclically relative to the seasonal variation in equity returns, though with a smaller magnitude of variation.[2] These empirical regularities are consistent with seasonally depressed investors favoring safe securities over risky securities during the periods when they suffer from the condition. In principle, variation in individuals' risk preferences over time and heterogeneity in risk preferences across individuals are not controversial notions, having been documented by Barsky, Juster, Kimball, and Shapiro (1997), for instance. We are interested in exploring whether a standard asset pricing model with seasonally varying preference parameters is capable of explaining, at both the qualitative and quantitative levels, the seasonal movements and co-movements in risky and risk-free returns.

Consideration of seasonality in preference parameters to capture seasonality in the macroeconomy, as in this paper, is not unprecedented. For example, Chatterjee and Ravikumar (1992) explore the impact of seasonal fluctuation in demand (with a Christmas peak) and in production (with a winter slowdown) on the fit of macro models, and Braun and Evans (1995) consider the degree of changes in consumption preferences, technology, and government purchases that are required to explain observed seasonal patterns in output and investment. Time preference and seasonal taste shift parameters have also been used to account for seasonally varying conditional covariances of consumption growth and excess returns. See, for instance, Ferson and Harvey (1992). While seasonal depression may appear to be a surprising component to include in a model of preferences, in character it closely resembles existing seasonal perturbations to tastes and technology in that

[1] Kamstra et al. (2003) find that daily realized stock returns are significantly lower than average during the fall season and significantly higher than average during the winter season, leading to higher-than-average holding-period returns for investors who hold stock through the fall and winter seasons.

[2] Additional papers have explored the influence of seasonal depression on other facets of financial markets, finding largely supportive results. Kamstra, Kramer, Levi, and Wermers (2011) investigate the flow of funds between safe and risky categories of mutual funds and find, controlling for other factors, that there are net flows out of risky funds and into safe funds in fall, and the patterns reverse in winter, consistent with the seasonal depression hypothesis. Garrett, Kamstra and Kramer (2005) explore seasonally varying risk aversion in an equilibrium asset pricing model which allows the price of risk to vary through the seasons, and they find evidence consistent with the seasonal depression hypothesis. DeGennaro, Kamstra, and Kramer (2008) study bid-ask spreads, Dolvin, Pyles, and Wu (2009) and Lo and Wu (2008) study analysts' stock earnings forecasts, Dolvin and Pyles (2007) study the underpricing of initial public stock offerings, Pyles (2008) studies returns to real estate investment trusts; all find evidence consistent with the influence of seasonal depression on markets. Dowling and Lucy (2008) enlarge Kamstra et al.'s (2003) original study to 37 countries and find similar results.

it also leads to refutable implications. Most importantly, the variation in preferences must be of sufficient magnitude to overcome the standard capital asset pricing result that risky and risk-free returns should move in tandem.

This paper contributes most closely to the literature that explores time-variation in expected returns and/or return volatility which arises due to time-varying preferences. Mehra and Sah (2002) find large changes in volatility can result from small changes in discount rates and risk aversion (where the discount rates may change, for instance, due to changes in the risk of catastrophe, and risk aversion may change due to psychological biases). Several models of time-varying risk aversion have focused on habit formation. Brandt and Wang (2003), for instance, consider a habit persistence model and a law of motion for log relative risk aversion whereby risk aversion is shocked with news about both consumption growth and inflation. Bekaert, Grenadier, and Engstrom (2010) consider stochastic risk aversion as a preference shock, motivated by a habit persistence framework. There is also a significant literature in asset pricing that studies the relationship between time-varying risk and risk aversion. One strand of the literature focuses on the empirical observation that equity premia seem to be higher in recessions than in booms (see, for instance, Fama and French (1989)). Campbell and Cochrane (1999) build a representative agent model and show that when the representative agent exhibits habit formation, his/her risk aversion is higher at business cycle troughs than it is at peaks. As a result, equity premia are higher at business cycle troughs than they are at peaks. Bansal and Yaron (2004) employ a model in which the representative agent has Epstein and Zin (1989) recursive preferences and show that when consumption growth has a small time-varying but persistent and predictable component the model can generate an equity premium close to what is observed in the U.S. data.

A second strand of the literature focuses on the observation that the equity premium seems to have declined over time, at least until recently. For instance, Lettau, Ludvigson, and Wachter (2008), using a regime-switching model, show that consumption volatility seems to have moved into a low-volatility regime and that the low observed equity premium can be rationalized by such regime changes in consumption volatility. A number of these papers, including Bansal and Yaron (2004) and Lettau, Ludvigson, and Wachter (2008), also exploit the notion of levered equity returns introduced by Campbell (1986) and further developed by Abel (1999). In contrast to these papers, our main results are based on a model that has no habit persistence or time-variation in risk but does incorporate levered equity and very simple preference shocks motivated by seasonal depression.[3,4] Similar to the entire literature on time-variation in expected returns which arises due to time-varying preferences, the expected return patterns we document cannot be arbitraged away, at least in this theoretical framework. Equity returns are never, in expectation, less than the risk-free return; there is no arbitrage opportunity. Agents holding risky assets are simply rewarded more, in expectation, in some periods than in others, to compensate them for bearing risk in periods when other agents prefer not to bear risk.

We start by investigating an equilibrium asset pricing model in which a representative agent's preferences vary across two seasons: one season has low risk aversion and the other has high risk aversion. We build on Shefrin's (2008, chapter 14) point that the risk preferences of heterogeneous investors can be captured through a representative agent model. We explore whether a reasonable set of values for key model parameters is capable of generating the empirically observed sign and magnitude of seasonal changes in equity and Treasury returns previously documented. In particular, we seek to match the following empirical regularities: (i) high risky asset returns during the season when the risk-free returns are low and low risky asset returns during the season when the risk-free returns are high, and (ii) much greater seasonal variation in risky asset returns than risk-free asset returns. We start with a simple setting, adapting the Mehra and Prescott (1985) model to allow for seasonal changes in risk aversion. In that simple setting, however, we have trouble reproducing the stylized fact that risk-free returns vary much more modestly than risky asset returns. Thus we turn to the Epstein and Zin (1989) model, adapting it to allow for seasonal cycles in both risk aversion and the intertemporal elasticity of substitution (IES), and find much more satisfactory results. The notion that the preference parameter IES may change over time is not controversial. See, for instance, Blundell, Browning and Meghir (1994), Attanasio and Browning (1995), and Atkeson and Ogaki (1996).

In the Epstein and Zin framework, with seasonally varying IES and seasonally varying risk aversion, we are able to match the stylized fact of counter-cyclical seasonal patterns in safe versus risky returns and we are able to closely match the magnitude of the returns across the seasons. Furthermore, a testable implication comes out of the model: for sensible parameter values that are not rejected by the data, the volatility of the risky asset returns must be higher in the seasons when equity returns are high, even though consumption risk is invariant over the seasons in this model.

[3] We focus our attention on *preference*-based models in large part due to experimental evidence (discussed more fully below) that depression influences investor risk preferences. One might alternatively opt to employ *beliefs*-based models in attempting to match the observed seasonal variation in returns. See Basak and Yan (2007, 2010) for an example where a belief-based model yields similar implications to a preference-based model.

[4] We employ the notion of pricing in the presence of *levered* equity because this allows us to match more easily the *level* of bond and equity returns, permitting us to explore how seasonal variation in risk preferences facilitates the model's ability to match seasonal variation in returns around the mean level returns.

Consistent with this implication from the model, we find risky asset volatility jumps up together with equity returns in the data. Further, consistent with the model calibration, consumption risk does not go up in the season with high equity returns. In the data, risk based on seasonally unadjusted consumption actually falls in that season, with the decline being economically and statistically significant.

A novel contribution of this paper is that seasonal changes in IES may help to explain the dynamics of risky and risk-free asset returns over the year. While Kamstra, Kramer, and Levi (2003) hypothesized that seasonal variation in risk aversion alone underlies the annual patterns they sought to explain, our findings suggest an important extension, namely that seasonal variations in agents' willingness to substitute consumption across periods may also play an important role.

The remainder of the paper is organized as follows. In Section 1 we outline the hypothesis under which seasonal depression may influence financial markets. In Section 2 we specify asset pricing models in which the representative agent exhibits preferences that vary with the seasonal timing of depression. In Section 3 we introduce the data used for the calibration exercises and discuss our results, including an overview of which cases lead to the matching of return magnitudes and/or counter-cyclical seasonal patterns in risky versus risk-free returns. In Section 4 we explore testable implications of the model regarding volatility. Piazzesi (2001) observes that there is seasonality in the cross-correlation of consumption growth and returns, and this seasonality may impact stock pricing. This leads her to question the use of seasonally adjusted data when testing consumption-based asset pricing models. Thus, in Section 5 we explore and reject the alternative explanation that seasonal variation in consumption growth accounts for the observed seasonal return patterns. We conclude in Section 6.

1 Seasonal Depression

The medical condition seasonal affective disorder (SAD) is a seasonal form of depression that affects a sizable fraction of the population; additional numbers suffer from the milder condition known as winter blues (sometimes called subsyndromal SAD).[5] Kasper et al. (1989b) explain

[5] As Mersch (2001) and Thompson et al. (2004) note, specific estimates of the prevalence of SAD and subsyndromal SAD vary considerably, depending on the diagnostic criteria and sample selection methods employed by the researchers. Rosen et al.'s study of a sample in Florida found the incidence of full-blown SAD was less than 2%. Schlager et al. (1995) found that 9% of their New York sample met the criteria for a diagnosis of SAD and an additional 29% had "significant" winter depressive symptoms without meeting the formal criteria for SAD diagnosis (the latter group would be described as having subsyndromal SAD). Schlager et al. reported that both groups suffered functional impairment associated with their seasonal condition. Rosen et al. (1990) drew a sample from various locations in the U.S. and found that the combined proportion of individuals who satisfy the criteria for either SAD or subsyndromal SAD was 17%. Kasper et al. (1989a) found the combined proportion was 18% in a sample drawn from Maryland. The nature, incidence, and cause of SAD are discussed in a wide range of articles in the medical and

that the distinction between SAD and subsyndromal SAD is not clear-cut; they recommend that both conditions be viewed along a continuum of seasonal depression. Marked seasonal variation in depression has even been documented among individuals whose symptoms are not extreme enough to be labeled as suffering from SAD or winter views. For instance, Harmatz (2000) demonstrates that depression varies across the seasons significantly even among people who do not suffer from SAD, peaking in the fall/winter seasons, especially for women. Kramer and Weber (2012) find a similar result, documenting that depression peaks in the fall/winter seasons even among healthy individuals.

Medical research has established that among the various possible environmental factors that might cause seasonal depression, length of daylight appears to be the primary cause.[6] Individuals who suffer from seasonal depression typically begin experiencing depression in the early fall and recover by late spring. (The peak point of onset across individuals is around September/October and the peak point of recovery is around March/April. We discuss this timing more fully in footnote 10.)

Previous research in psychology has established a link between depression and increased risk aversion, including risk of a financial nature.[7] With a substantial fraction of the population experiencing seasonal depression in the fall and winter months, Kamstra et al. (2003) conjecture that the proportion of risk-averse investors is higher in those seasons. Risk-averse investors, they argue, begin to shun risky stocks in the fall as the length of day shortens, which has an immediate negative influence on stock prices, contributing to lower contemporaneous returns and higher expected future returns. As the amount of daylight rebounds through the winter months, investors begin to recover from their depression and become more willing to hold risky assets, which, Kamstra et al. (2003) posit, has a positive influence on stock prices, contributing to higher contemporaneous returns and lower expected future returns.

To better understand the implications for seasonal patterns in expected risky returns, consider an investor who purchases stock around the time when risk averse investors have driven prices to an autumn low. S/he will face higher expected future returns over the semi-annual holding

psychology literatures that is surveyed by Lee et al. (1998).

[6]See, for instance, Molin et al. (1996) and Young et al. (1997).

[7]Depressed individuals have been shown to be significantly more risk averse than non-depressed individuals. See Zuckerman (1984) and Carton et al. (1995), Kramer and Weber (2012), among others. Further, certain standard psychological measures of risk aversion have been shown to capture *financial* risk aversion. See, for instance, Harlow and Brown (1990) and Horvath and Zuckerman (1993). See Kamstra, Kramer, and Levi (2003) and Kamstra, Kramer, and Levi (2012) for more details on these links. Additionally, Kramer and Weber (2012) study hundreds of individuals across the seasons, including individuals who suffer from seasonal depression and individuals who do not. They find the depressed group is more averse to financial risk than the non-depressed group in all seasons, most markedly so in fall/winter.

period that lasts until around the point in the spring when most of the SAD-influenced investors have rebounded to their previous (lower) level of risk aversion and have resumed risky holdings. Similarly, an investor who purchases stock at the point in the year when previously risk averse investors are resuming risky holdings will face relatively lower expected future returns over the holding period that lasts until the annual cycle in risk aversion begins anew. Overall, security returns, which are a "flow" quantity, respond to the "flow" of SAD-affected investors (i.e., the change in the incidence of seasonal depression in the investing population).

Kamstra et al. (2003) demonstrate economically and statistically significant seasonal patterns in international stock indices consistent with this intuition. The patterns are more prominent in stock markets at extreme latitudes, such as Sweden, where the fluctuations in daylight are more extreme. Further, in southern hemisphere markets such as Australia, the seasonal patterns are six months out of phase relative to the northern hemisphere markets, just as the seasons are.

Kamstra, Kramer, and Levi (2011) document a counter-cyclical seasonal pattern in Treasury note and Treasury bond returns relative to stock returns, consistent with seasonally varying risk aversion being the underlying force behind both effects. If seasonally depressed investors are shunning risky stocks in the fall as they become more risk averse, then they should be favoring safe assets at that time, which should lead to an offset seasonal pattern in Treasury returns relative to stock returns. Kamstra, Kramer, and Levi (2011) demonstrate that the seasonal pattern in Treasury returns does not arise due to any of a wide range of alternative cyclical factors, including macroeconomic risks, Treasury market liquidity, Fama and French (1993) risk factors, Baker and Wurgler (2006) sentiment, and cross-market hedging, among others.

In addition to its association with elevated risk aversion, there is another feature of depression which may have important implications for financial markets. Research in psychology has established that depressed individuals tend to exhibit greater preference to consume in the present, which manifests in a condition termed "compulsive buying disorder." Lejoyeux, Tassain, Solomon, and Adès (1997) and Lejoyeux, Haberman, and Adès (1999) find that the incidence of compulsive buying among depressed individuals is about 40%, compared to an incidence in the overall U.S. population around 6%. (See Koran, Faber, Aboujaoude, Large, and Serpe (2006).) The most common theory of compulsive buying is that low serotonin levels found in depression, including SAD, are associated with increased rates of impulsivity. It may be that individuals who suffer from compulsive buying disorder use the experience as a way to self-medicate their condition, as Lejoyeux et al. (1997) find that treatment with antidepressants, including serotonin re-uptake inhibitors (SSRIs), can help to decrease the frequency and severity of uncontrolled buying. Compulsive buying disorder is characterized by a lack of impulse control over making purchases and hence smoothing consumption over time, or stated in terms more familiar to financial economists, low intertemporal elasticity of substitution. Berns, Laibson, and Loewenstein (2007) find that IES is impacted by emotion and affect. Recall that SAD is itself a condition associated with seasonal changes in affect; specifically depression. Thus, just as Kamstra, Kramer, and Levi (2003) argue that seasonality in depression implies seasonality in risk aversion, so we hypothesize that seasonality in depression may also manifest itself in seasonally varying IES. We explore the implications more fully below.

2 Asset Prices and Returns

As noted above, SAD and winter blues are seasonal forms of depression that influence a substantial fraction of the population, perhaps everyone to some degree according to some research. Seasonal cycles in risky versus safe returns have been observed that temporally coincide with seasonally depressed individuals experiencing seasonally varying risk aversion: expected risky returns are higher during high risk aversion seasons and expected risk-free returns are lower. While the aggregate market data support the seasonal depression hypothesis, to date no attempt has been made to see whether there exists a reasonable parameterization of an asset pricing model capable of generating the expected seasonal patterns in safe versus risky asset returns, or whether such a parameterization is capable of generating the magnitude of changes in returns that are observed in these securities. In this section, we develop a model for the purpose of examining that possibility.

It may seem most natural to consider a model with heterogeneous agents, some of whom suffer from seasonal depression and others who do not. In such a model, returns would vary across seasons such that all assets are held at all times, in the spirit of the CAPM. However, when the market is complete, the aggregation theory of Constantinides (1982) shows that a representative agent model applies without loss of generality as far as equilibrium asset prices are concerned. Preferences within a representative agent economy are a weighted average of the preferences of agents within a heterogeneous agent economy. As the preferences of agents who experience seasonal depression change over time, the preferences of the representative agent (again, which can be thought of as the weighted average of the preferences of all agents in the economy) will also vary seasonally. Thus in our model we assume that the representative agent has seasonally varying preferences. In addition, we assume that the representative agent has recursive utility as in Epstein and Zin (1989).[8] As we

[8]Note that we do not assume that in a heterogeneous agent economy the agents would have recursive utility. In general, when heterogeneous agents have recursive utility, the aggregate representative agent may not have recursive utility. Our assumption that the representative agent has recursive seasonally varying utility is for its analytical convenience.

show later, if we employ a seasonally varying risk aversion parameter in a standard intertemporally additive expected utility function, the resulting economy is not stationary. Recursive utility, in contrast, is more flexible and ultimately more successful in modeling seasonally varying preference parameters.

In short, our final model differs from a a standard Lucas (1978) model in two important respects. First the representative agent has recursive utility as in Epstein and Zin (1989), and second the parameters of the agent's utility function are seasonally varying. Let us start by considering the simple Lucas (1978) model.

2.1 A Lucas Model with Seasonally Varying Risk Aversion

To illustrate the basic intuition of our theory and to motivate the use of recursive preferences, we start with the Lucas (1978) economy and modify it to allow the representative agent to have seasonally varying preferences. The representative agent is assumed to have standard intertemporal expected utility,

$$U_0 = E_0\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\gamma_t}-1}{1-\gamma_t}\right],$$

where γ_t is the level of risk aversion which varies seasonally. We assume that the level of risk aversion varies cyclically and deterministically between a low risk-aversion season and a high risk-aversion season.

This simple economy can be used to illustrate the potential impact of seasonally varying risk aversion on asset returns. It is straightforward to show that the asset returns in the economy must satisfy

$$0 = E_t\left[\frac{c_{t+1}^{-\gamma_{t+1}}}{c_t^{-\gamma_t}}(R_{M,t+1}-R_{f,t})\right], \tag{1}$$

where $R_{M,t+1}$ is the return on the risky asset and $R_{f,t}$ is the return on the risk-free asset.[9]

We see from this equation that when risk aversion changes over time, the ratio by which we multiply the excess return varies from period to period even if consumption patterns remain the same across periods. To satisfy Equation (1), excess returns need to change in response. Thus this simple economy exhibits the seasonally varying return cycles that we aim to study.

[9] Note that the price of asset a is measured at the beginning of a time period (season) t as $P_{a,t}$. Dividends paid on the asset from the beginning of period t to the beginning of period $t+1$ are $d_{a,t+1}$. The realized return on the asset, held from the beginning of time t to the beginning of time $t+1$, is $R_{a,t+1} = (P_{a,t+1} + d_{a,t+1} - P_{a,t})/P_{a,t}$.

While it is straightforward to use this simple economy to illustrate the potential impact of seasonally varying risk aversion on asset returns, the equilibrium has some undesirable properties. To see this, consider the risk-free rate in this economy. The Euler equation for the risk-free asset is given by

$$1 \quad = \quad \beta E_t \left[\frac{c_{t+1}^{-\gamma_{t+1}}}{c_t^{-\gamma_t}} (1 + R_{f,t}) \right]. \tag{2}$$

Let the consumption growth be given by $\frac{c_{t+1}}{c_t} = g_{t+1}$. Then Equation (2) can be written as

$$\frac{1}{1 + R_{f,t}} = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma_t} c_{t+1}^{\gamma_t - \gamma_{t+1}} \right] = \beta c_t^{\gamma_t - \gamma_{t+1}} E_t \left[g_{t+1}^{-\gamma_{t+1}} \right]. \tag{3}$$

It is evident from Equation (3) that the risk-free rate will vary cyclically over time due to the cyclical nature of the risk aversion, and if consumption is non-stationary (growing, for instance), then the risk-free rate will also exhibit the undesirable feature of non-stationarity. However, even in the case of invariant risk preferences, when the consumption process is non-stationary with non-trivial growth it is a challenge to adopt the standard Lucas (1978) model to study the equilibrium. Mehra (1988) shows that when the representative agent has CRRA utility with constant relative risk aversion and the consumption growth rate follows a stationary Markov process, the equilibrium asset returns will be stationary. Unfortunately, this special-case result does not extend to seasonally varying preferences such as we study. When consumption (c_t) grows, it follows from Equation (3) that $R_{f,t}$ oscillates with greater and greater magnitude and thus is not stationary. It is readily shown that the return on the market portfolio is also non-stationary.

2.2 Recursive Utility with Seasonal Depression

In order to resolve the non-stationarity problem, we turn to recursive preferences as developed by Epstein and Zin (1989), modified to allow for a representative agent with seasonally varying risk aversion. We assume that the representative agent's preferences are captured by

$$U_t = \left[c_t^{(1-\gamma_t)/\theta_t} + \delta (E_t U_{t+1}^{1-\gamma_t})^{1/\theta_t} \right]^{\theta_t/(1-\gamma_t)} \tag{4}$$

where $\theta_t = (1 - \gamma_t)/(1 - 1/\psi_t)$. Here ψ_t is the intertemporal elasticity of substitution (IES). As discussed in Section 1, previous research in psychology has suggested connections between depression and greater risk aversion and between depression and lower IES. Thus, during the seasons when an individual suffers from seasonal depression, we expect his/her IES, ψ_t, will be lower, consistent with his/her being less willing to substitute his/her consumption today for consumption tomorrow. Assuming $\gamma_t > 1$, the lower ψ_t is associated with a lower θ_t. At the same time, the seasonal depression leads to higher risk aversion, γ_t. Overall, the effect of seasonal depression on this individual is a higher γ_t and a lower θ_t. Conversely, in seasons when the individual is not depressed, his/her behavior can be characterized by a lower γ_t and a higher θ_t. When $\theta_t = 1$ and γ_t is a constant, the result is the standard intertemporally additive expected utility. Following Bansal and Yaron (2004), we assume that $\psi_t > 1$, implying that $\theta_t < 0$.

With this model, the utility maximization problem of the representative agent is

$$J_t(W_t, x_t; \gamma_t, \theta_t) = \max_{c_t, \pi_t} \left[c_t^{(1-\gamma_t)/\theta_t} + \delta (E_t J_{t+1}^{1-\gamma_t})^{1/\theta_t} \right]^{\theta_t/(1-\gamma_t)}$$

subject to the constraint

$$W_{t+1} = (W_t - c_t)\pi_t(1 + R_{t+1})$$

where W_t is the wealth of the agent, x_t is the vector of state variables, π_t is the vector of portfolio weights, and R_t is the vector of asset returns (consisting of the risky market return, $R_{M,t}$, and the risk-free return, $R_{f,t}$).

We assume the agent's preferences oscillate across periods. In one period the agent has low risk aversion and high IES, in the next s/he has high risk aversion and low IES, and so on. We should emphasize that the representative agent knows his/her preferences will change over time and all of his/her decisions take that fact into account. That is, the representative agent is not myopic.

By an argument similar to that of Epstein and Zin (1989), it can be shown (see Equation (21) in Appendix A) that

$$P_{a,t} = \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t \left[g_{t+1}^{-\gamma_t} (w_{t+1}(x_{t+1}) + 1)^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})} - 1} (P_{a,t+1} + d_{a,t+1}) \right], \tag{5}$$

where $P_{a,t}$ is the price of asset a at time t, $d_{a,t}$ is the dividend on asset a at time t, and $w_t(x_t)$ is the price-dividend ratio of the market portfolio at time t. We use this expression to represent the price of the risky market portfolio, $P_{M,t}$, and the price of the risk-free asset, B_t.

Equation (5) implies (see Equation (18) in Appendix A) that the price-dividend ratio of the market portfolio satisfies

$$w_t(x_t) = \delta \left(E_t \left[\left(g_{t+1} (1 + w_{t+1}(x_{t+1}))^{\frac{\theta_{t+1}}{(1-\gamma_{t+1})}} \right)^{1-\gamma_t} \right] \right)^{1/\theta_t}. \tag{6}$$

With the help of Equation (6), we can gain some insight into how seasonal depression influences market returns in our model. First we establish a non-cyclical benchmark: replace γ_t with γ_{t+1} and θ_t with θ_{t+1} so that the representative agent's level of depression (or non-depression) is the same across all periods. In that case, Equation (6) reduces to

$$w_t(x_t) = \delta \left(E_t \left[g_{t+1}^{1-\gamma_{t+1}} (1 + w_{t+1}(x_{t+1}))^{\theta_{t+1}} \right] \right)^{1/\theta_{t+1}} .$$

Then, because the γ and θ parameters do not vary across periods in the non-cyclical case, the price-dividend ratio does not vary systematically across periods in the non-cyclical case.

Now let us contrast the non-cyclical benchmark with the cyclical case, in which the representative agent cycles back and forth between periods of experiencing seasonal depression and not experiencing seasonal depression. To start with, consider what happens to the price-dividend ratio in the period when the agent suffers from seasonal depression in period t; i.e., $\gamma_t > \gamma_{t+1}$ and $\theta_t < \theta_{t+1}$. Given that the term within the round brackets in Equation (6) is greater than one (which we assume), the expectation is less than one for $\gamma_t > 1$. Thus the price-dividend ratio during the seasonally depressed period is smaller than the price-dividend ratio in the benchmark case. Similarly, when the agent does not suffer from seasonal depression in period t ($\gamma_t < \gamma_{t+1}$ and $\theta_t > \theta_{t+1}$), the price-dividend ratio during the non-seasonal-depression period is larger than that in the benchmark case. That is, when the preference parameters change cyclically, the price-dividend ratio moves counter-cyclically relative to risk aversion.

The intuition behind the behavior of the price-dividend ratio as risk aversion varies is as follows. Risk aversion is higher in the depression season and the price of the risky asset $P_{M,t}$ is, as a result, lower than it would be otherwise. Consequently, the price-dividend ratio $P_{M,t}/d_{a,t}$ is smaller, resulting in a counter-cyclical relationship between risk aversion and the price-dividend ratio. The intuition behind the relationship between the price-dividend ratio and IES can be demonstrated first by recalling that the general model we employ nests the standard model where IES equals the reciprocal of the coefficient of relative risk aversion (that is, IES and risk aversion are inversely related). Thus, as IES drops, the price-dividend ratio drops. More generally, as IES drops, an agent becomes less willing to delay consumption. In order to convince such an agent to forgo consumption in favor of buying a security, the price of that security must fall. Hence with the lower IES comes a lower price-dividend ratio.

We turn now to consider the market return. The return on the market portfolio is related to the price-dividend ratio through

$$R_{M,t+1} = \frac{P_{M,t+1} + d_{M,t+1}}{P_{M,t}} = \frac{(w(x_{t+1}) + 1)g_{t+1}}{w(x_t)}. \tag{7}$$

Taking the expectation yields

$$E_t[R_{M,t+1}] = E_t\left[\frac{P_{M,t+1} + d_{M,t+1}}{P_{M,t}}\right] = \frac{E_t[(w(x_{t+1}) + 1)g_{t+1}]}{w(x_t)}. \tag{8}$$

It is evident from Equation (8) that as the representative agent's preferences move back and forth between the seasonal depression and non-seasonal-depression seasons, the expected market return moves pro-cyclically relative to risk aversion. That is, the expected market return is higher in the seasons when agents experience seasonal depression.

Next we consider the risk-free rate of return. Equation (5) implies that the price of the one-period risk-free asset satisfies

$$B_t(x_t) \quad = \quad \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[g_{t+1}^{-\gamma_t}(w_{t+1}(x_{t+1}) + 1)^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}-1}\right]. \tag{9}$$

The impact of a change in risk aversion on bond prices has been studied by Mehra and Prescott (1985) for the case of intertemporally additive expected utility. Basically, whether an increase in risk aversion leads to an increase in bond prices depends on the distribution of consumption growth, g_{t+1}. For the parameters calibrated in their paper, an increase in risk aversion increases bond prices and reduces expected bond returns, as intuition would suggest. Similar logic applies here: when risk aversion rises, the result is a lower expected risk-free return. The impact of a decrease in θ_t on the bond price does not depend on the distribution of consumption growth, however. When θ_t decreases, the bond price increases as long as $w_t(x_t) > 1$, and the result is a lower expected risk-free return. Overall, the risk-free rate moves counter-cyclically relative to risk aversion; it is lower in seasons when agents suffer from seasonal depression.

In summary, when the representative agent has seasonally varying preferences, conditional expected market returns and risk-free rates exhibit counter-cyclical seasonal patterns relative to each other. In addition to its implications for seasonal return cycles, the theory has implications for seasonal volatility cycles. Equation (7) implies that the volatility of the market return is greater in periods when the representative agent experiences greater risk aversion. This is an implication that, to the best our knowledge, has not been examined elsewhere in the literature. In Section 4 we discuss the intuition behind this result and explore its empirical validity through calibration.

2.3 A Simple Mehra-Prescott-Style Model with Seasonally Varying Preferences

Recall we assume that during seasons when the representative agent has high risk aversion, s/he has low IES (both being individually associated with increased depression as discussed in Section 1), and that the agent cycles between these two sets of preferences. Now we further assume that each of the periods (seasons) is six months in length.[10] During one 6-month period (season) every year the set of preference parameters is $\{\gamma_H, \theta_H\}$ and in the other 6-month season of every year, the set is $\{\gamma_L, \theta_L\}$. The parameter set $\{\gamma_H, \theta_H\}$ corresponds to the time period when the agent suffers from seasonal depression and consequently exhibits high (H) risk aversion. At the same time, we assert that the agent experiences low IES (low θ). In the other 6-month period of every year, the agent exhibits low (L) risk aversion and high IES (high θ).

Following Mehra and Prescott, let us assume a two-state Markov world and that $P(i,k,d) = w_i^k d$ where $i = 1,2$ indexes states. We use $k = H, L$ to index preference parameters across the high and low risk aversion seasons. Equation (6) then yields (see Equations (19) and (20) in Appendix A) the following system of equations for price-dividend ratios,

$$(w_i^H)^{\theta_H} = \delta^{\theta_H} \sum_{j=1}^{2} \phi(i,j) g_j^{1-\gamma_H} (1 + w_j^L)^{\frac{\theta_L(1-\gamma_H)}{(1-\gamma_L)}} \tag{10}$$

$$(w_i^L)^{\theta_L} = \delta^{\theta_L} \sum_{j=1}^{2} \phi(i,j) g_j^{1-\gamma_L} (1 + w_j^H)^{\frac{\theta_H(1-\gamma_L)}{(1-\gamma_H)}}. \tag{11}$$

Here w_i^k has the interpretation of price-dividend ratios when the preference parameters are γ_k and θ_k, the current state of consumption growth is i, and ϕ is the Markov state-transition probability matrix.

In the case of a two-state Markov world and two sets of preference parameters, it follows from Equation (9) (see Equations (23) and (24) in Appendix A) that the one-period risk-free asset price

[10] The periodicity we adopt in modeling the influence of seasonal depression is based on clinical studies by Young et al. (1997) and Lam (1998) who document the timing of the clinical onset of and recovery from SAD symptoms among North Americans known to be affected. They find that most people who suffer from SAD experience their symptoms for about six months during the fall and winter seasons, with Lam (1998) finding the peak in onset during October and the peak in recovery during April (he reports monthly statistics) and Young et al. (1997) finding the peak in onset in the second week of October. (Young et al. report weekly data but do not report recovery statistics.) Thus, for the sake of parsimony, we opted to develop a two-season model based on the representative agent suffering from seasonal depression during the fall and winter (beginning in October, consistent with the timing observed by these clinical researchers) and not suffering from seasonal depression for the other six months of the year (beginning in April). (In a previous version of this paper, we performed robustness checks where the six-month seasons begin in September and March and we found similar results.) In principle one might consider developing instead a more complex model with four distinct seasons (fall, winter, spring, and summer).

satisfies the following system of equations:

$$B_i^H = \delta^{\theta_H}(w_i^H)^{1-\theta_H}\sum_{j=1}^{2}\phi(i,j)g_j^{-\gamma_H}(1+w_j^L)^{\frac{\theta_L(1-\gamma_H)}{(1-\gamma_L)}-1}, \tag{12}$$

$$B_i^L = \delta^{\theta_L}(w_i^L)^{1-\theta_L}\sum_{j=1}^{2}\phi(i,j)g_j^{-\gamma_L}(1+w_j^H)^{\frac{\theta_H(1-\gamma_L)}{(1-\gamma_H)}-1}. \tag{13}$$

We solve this system of equations analytically for equilibrium values of w_i^k and B_i^k. Then we integrate over growth states to determine the expected risky and risk-free rates of returns in the high and low risk aversion periods.

It is apparent from these expressions that risk-free rates are stationary when the representative agent has Epstein and Zin (1989) utility, unlike the case of intertemporally additive expected CRRA utility. We should note that even if $\theta_t = 1$, that is when intertemporal elasticity of substitution is equal to the inverse of risk aversion, the case of Epstein and Zin preferences with seasonal depression does not reduce to intertemporally additive expected utility with seasonal depression, unlike the case without seasonal depression. This is because when risk aversion is time-varying, the recursive formulation of expected utility is not equivalent to intertemporally additive expected utility. The equivalence arises only when the risk aversion parameter is not time-varying. The recursive utility in Equation (4) is homogeneous while the intertemporally additive expected CRRA utility is not. Therefore in dealing with a consumption process with stationary non-zero growth rates, recursive utility in Equation (4) leads to a stationary return process while the intertemporally additive expected CRRA utility does not.[11]

3 Data and Model Calibration

We first produce stylized facts on return seasonalities. We perform all of our analysis on a semi-annual periodicity so that a semi-annual risk-free rate, set at the beginning of the semi-annual period, is the appropriate quantity to calibrate to our model's risk-free rate. Although a shorter-term instrument, such as the 1-month T-bill, could be rolled over to produce a 6-month return (and indeed we consider such a return for robustness), this is not a true 6-month risk-free rate because the 1-month bill varies unpredictably month-to-month over the semi-annual period. Unfortunately, 6-month T-bill rates are not available until 1959, thus our calibration exercise relies on data from

[11] We should emphasize that the failure of CRRA to emerge as a special case of the Epstein-Zin model when preferences vary due to seasonal depression is *not* a consequence of myopia in any sense. The representative agent is well aware that his/her preferences change over time and accounts for that fact in his/her decisions at all points in time.

1959 onwards. Similarly, our calibration exercise employs only data prior to 2008, in order to prevent the financial crisis from appearing to drive our results. (The crisis included a market crash and an unprecedented flight on the part of investors into the safe haven of Treasury securities in the fall of 2008, right around the break between the semi-annual periods we consider.) However, to investigate subsample instabilities, we also present summary statistics on data beginning as early as 1926 and extending to the end of 2010. Summary statistics on samples pre-1959 rely on the 1-month T-bill rolled over to form a 6-month return, as a stand-in for the 6-month bill.

3.1 Data

In order to get an overview of equity and Treasury market seasonalities, we present Table I, containing average nominal 6-month returns for equities (based on the CRSP equal-weighted U.S. total-market index including dividends), average nominal 6-month returns for risk-free securities (U.S. Treasury bills), and equity premium values over the period 1926-2010 and various sub-samples. We consider inflation adjusted T-bill and (value-weighted) equity returns in Table II. Our choice of series warrants some discussion. First, the use of a total-market equity index ensures the inclusion of small, risky stocks, which are perhaps relatively more likely to exhibit seasonally varying returns due to possibly seasonally varying investor risk aversion. Further, the consideration of an equal-weighted equity index places more weight on the small, risky stocks relative to a value-weighted index.[12] For comparison, we also consider value-weighted equity returns, below. Second, for the risk-free security, we consider the return to holding a 6-month T-bill, where available (i.e., starting in 1959), and the return to holding a 30-day T-bill (rolled over monthly) before 1959.

The statistics we present are based on splitting the year into October-March and April-September 6-month seasons. This split coincides with the clinical observation that October is the approximate point when there is a peak in SAD diagnoses and April is the approximate peak point of recovery from SAD, as reported in footnote 10 above. In a previous version of this paper we performed a robustness check based on splitting the year into September-February and March-August seasons and found similar results.

The first noteworthy feature of Table I is that in virtually every panel the average equity return is much higher in the fall/winter season (October-March) than in the spring/summer season (April-September), significantly so in most cases.[13] During the full 1926-2010 sample period, the

[12]See Kamstra et al. (2003, 2012) and Dowling and Lucy (2008) for further discussion of the use of equal-weighted versus value-weighted equity returns in testing for the influence of seasonal depression on returns.

[13]The simplest test for variation over the seasons would use the standard deviation of the semi-annual returns directly. However, this would ignore information about the cross-sectional variability of Treasury and equity returns and would not control for heteroskedasticity and autocorrelation. For the two longest samples, 1926-2010 and 1959-2007, we form a system of equations with the Treasury returns series and three size-sorted equity portfolio returns, thus taking advantage of cross-sectional information in returns. The three size-sorted portfolios make use of CRSP

Table I
Average Realized U.S. Nominal Returns
(6-Month Rates of Return)

Period	Season	EW Equity Return (%)	Risk free Rate (%)	Equity Premium (%)
Panel A	April-September	6.981	1.906	5.075
1926-2010	October-March	10.21	1.932	8.278
	Seasonal Change	3.230***	0.026	3.203***
Panel B	April-September	3.277	2.899	0.378
1959-2007	October-March	11.93	2.931	8.999
	Seasonal Change	8.654***	0.032	8.622***
Panel C	April-September	18.74	0.652	18.08
1926-1939	October-March	2.025	0.664	1.361
	Seasonal Change	-16.7**	0.012	-16.7**
Panel D	April-September	5.008	0.210	4.797
1940-1949	October-March	13.44	0.227	13.21
	Seasonal Change	8.431	0.017	8.414
Panel E	April-September	6.246	0.927	5.318
1950-1959	October-March	11.12	1.059	10.06
	Seasonal Change	4.876**	0.132*	4.743**
Panel F	April-September	3.095	2.016	1.079
1960-1969	October-March	12.56	2.200	10.36
	Seasonal Change	9.460***	0.184*	9.276***
Panel G	April-September	0.553	3.269	-2.72
1970-1979	October-March	13.15	3.594	9.557
	Seasonal Change	12.60***	0.326*	12.27***
Panel H	April-September	6.199	4.779	1.420
1980-1989	October-March	10.47	4.524	5.949
	Seasonal Change	4.273	-.26	4.528
Panel I	April-September	4.729	2.614	2.115
1990-1999	October-March	14.65	2.514	12.13
	Seasonal Change	9.919	-.10	10.02
Panel J	April-September	1.781	1.706	0.075
2000-2007	October-March	9.517	1.640	7.876
	Seasonal Change	7.736**	-.07	7.802**

Note to Table I: We report the average (nominal) 6-month equity returns and risk-free returns for the 1926-2010 sample and various sub-periods, including subsamples that omit the financial crisis period of 2008-2010. We consider the 6-month T-bill where available (1959 to the present), and the return to holding a 30-day T-bill rolled over each month otherwise. The equity return data are CRSP U.S. total-market (NYSE, NASDAQ, and AMEX) equal-weighted returns including dividends. The Treasury data are from the CRSP Fama Treasury Bill Term Structure Files. For each period or sub-period, we consider average returns for October through March, the average returns for April through September, and the equity premium (i.e., the difference between these two rates of return). Significance of the tests for seasonal difference in rates are indicated as follows: one, two, and three asterisks represent significance at the 10%, 5%, and 1% respectively, based on two-tailed tests, calculated as described in footnote 13.

equity premium investors earned was on average an astonishing 300 basis points higher during the fall/winter versus the spring/summer. Since the Depression, the decade with the largest seasonal difference in the equity premium was the 1970s, exceeding 12%. The only sub-sample during which the average equity return is not higher in the fall/winter season than in the spring/summer season is the first fourteen years of the data (1926-1939), shown in Panel C of Table I. The 1926-1939 period is, of course, a remarkable period of time, encompassing the bubble of the late 1920s and the Great Depression of the 1930s. The spring/summer of 1933 saw a return of 157% to the equal-weighted CRSP index (and while not included in the table, the value-weighted index experienced a return of 73% during the same period). Without including the 1933 data, the spring/summer average return shown in Panel C would be roughly 9% instead of about 19%. Six of the ten largest declines and five of the ten largest increases over the 85 years of the sample occurred in the 1930's. This extreme volatility suggests that this period is not characteristic of the equity market, and it bears noting that even the classic Mahra and Prescott (1985) paper documented substantial sub-sample variability in the equity premium, with an equity premium over 15% during 1919-1928, 0.18% during 1929-1938, 9% during 1939-1948, and over 18% during 1949-1958 (see their Table 1).

The second noteworthy feature of Table I pertains to the average nominal Treasury-bill returns. We consider very short maturity (6-month) Treasury securities and we find no strong seasonal return pattern at this maturity. We should emphasize that Kamstra, Kramer, and Levi (2011) find an economically large and statistically significant seasonal pattern in longer-maturity Treasury notes and bonds and find some evidence in the shorter end, trailing off monotonically as maturity shortens. They find Treasury returns are significantly lower than average during the fall/winter season when risky equity returns are higher-than-average, and vice versa for the spring/summer. Kamstra et al. focus their analysis on notes and bonds because monetary policy aims explicitly to remove seasonality in the money supply and is known to have a large moderating influence on the

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value-weighted decile returns, equal-weight averaging deciles 1 through 3 to form the first portfolio, deciles 4 through 7 to form the second portfolio, and deciles 8 through 10 to form the third. With these equity returns and the Treasury-bill returns we estimate a fixed-effects model with a dummy variable for the fall-winter season. Consistent with the typical implementation of a fixed-effects model, we allow each series to have a different mean, while estimating one set of parameter values for the variables each series has in common, in this case the dummy variable for fall-winter. We allow the dummy variable to have different coefficient estimates for the Treasury-bill series versus the equity series. From this regression we obtain the standard errors on the dummies to produce t-tests on the mean difference in semi-annual returns being different from zero. For the shorter decade-long data periods, it is not possible to reliably estimate this large a system of equations (see Foerster and Ferson, 1994, for instance), so for these sub-samples we estimate a system of two equations, regressing Treasury and CRSP equal-weighted returns on a constant and a dummy variable for the fall-winter season. For both sets of estimations we use Hansen's (1982) generalized method of moments (GMM) and Newey and West (1987, 1994) heteroskedasticity and autocorrelation consistent (HAC) standard errors. This estimation approach yields robust t-tests on the mean difference in the semi-annual returns. An identical process is used for calculating the t-test on the mean difference in the equity premium. Following Newey and West (1994) we use the Bartlett kernel and an automatic bandwidth parameter equal to the integer value of $4(T/100)^{2/9}$, where T is the number of observations available. For instruments for the GMM estimation we use the constant, the fall-winter dummy variable, and two lags of each dependent variable. Ferson and Foerster (1994) evaluate the small sample performance of GMM and HAC in a systems equation setting, based on monthly U.S. Treasury and stock returns. They show little or no bias of GMM estimation in this context.

shorter end of the term structure.[14] Thus while our consideration of very short maturity Treasury securities leads to the finding of little seasonal variation in Treasury returns (consistent with the Federal Reserve's monetary policy objectives), the reader should keep in mind the more general finding of significant seasonal variation in Treasury notes and bonds, with significant counter-cyclical seasonal variation in Treasury returns relative to that found in equity returns. In fact, we observe counter-cyclical seasonal variation in Treasury versus equity returns in Table II below, where we consider inflation-deflated returns.

The consideration of *real* (deflated) returns forms the basis for the rest of our analysis and that of other related studies, including Mehra and Prescott (1985). Table II contains real returns for equities and risk-free securities over the period 1959-2007 and for a few sub-samples and alternative seasonal breaks in the data. We do not report equity premia because the variation in premia are virtually identical to the variation in equity returns. Beyond the adjustment for inflation, there are two additional notable differences between Tables I and II.

First, in Table II we restrict our attention to data starting in 1959. This is due to availability of the risk-free 6-month Treasury yield data as well as the consumption data that we employ for our calibration exercise, described below. Because the seasonality in returns is very similar across sub-periods and the entire span of the data for virtually every case we consider, this restriction is not material to the calibration exercises we perform. Second, we make use of value-weighted returns in Table II rather than the equal-weighted returns we use in Table I. The convention in the asset-pricing literature when performing calibration exercises is to consider value-weighted returns, as they better capture total returns to the typical investor.

For the risk-free security, we present both the return to holding a 6-month T-bill and the return to holding a 30-day T-bill rolled over monthly. Panels B and C display results for the first half (1959-1983) and the last half (1984-2008) of our full sample period to facilitate the evaluation of sub-period stability.

Consider the summary statistics for the entire sample, shown in Panel A of Table II. Equity returns are remarkably larger in the fall/winter season, with statistically strong and economically meaningful variation across the two seasons.[15] Risk-free returns, whether measured by the return to holding the 6-month T-bill to maturity or the return to holding the 30-day T-bill rolled over monthly, are for the most part roughly 10 basis points lower in the fall/winter season than in the spring/summer season, though this difference is not statistically significant. The seasonal

[14] Gibson (1970), referring to Treasury bill rates, notes that an "aim of the Federal Reserve System is to accommodate seasonal swings in the financial needs of trade, and the System tries to do this by removing seasonal fluctuations from interest rates" (p. 442).

[15] Once again, we take advantage of the cross-sectional information in returns and form a system of equations using the Treasury returns series and three size-sorted equity portfolio returns, identical to that described in footnote 13 for the two longest samples, 1926-2010 and 1959-2007.

Table II
Average Realized U.S. Real Returns Over 1959-2007
(6-Month Rates of Return)

Period	VW Equity Return (%)	Risk-free Rate (6-Month Maturity, %)	Risk-free Rate (30-Day Maturity, %)
Panel A: 1959-2007			
April-September	1.135	0.939	0.673
October-March	6.442	0.938	0.615
Seasonal Change	5.31***	-.001	-.06
Panel B: 1959-1983			
April-September	-0.01	0.789	0.535
October-March	5.357	0.844	0.507
Seasonal Change	5.37***	0.055*	-.03
Panel C: 1984-2007			
April-September	2.328	1.103	0.825
October-March	7.571	1.012	0.705
Seasonal Change	5.24***	-.09**	-.12***

Note to Table II: We report the average (real) 6-month equity and risk-free returns for the full 1959-2007 sample and various sub-periods. We consider two distinct risk-free returns series: one based on the return to holding a 6-month T-bill, and another based holding a 30-day T-bill rolled over each month to form a 6-month semi-annual return. The equity return data are CRSP value-weighted returns including dividends, deflated using realized inflation as described in footnote 16. The Treasury data are from the CRSP Fama Treasury Bill Term Structure Files, deflated using predicted inflation as described in the text. For each sample period we consider average returns and the difference in average returns across the two seasons. Significance of the tests for seasonal difference in rates are indicated as follows: one, two, and three asterisks represent significance at the 10%, 5% and 1% respectively, based on two-tailed tests, calculated as described in footnote 15.

variation in Treasury yields is less stable than seen for equities, with yields tending to decline in the fall, but not uniformly across sub-periods, consistent with the evidence presented in Table I. One interpretation of the joint findings (based on our Table I and Table II and Kamstra, Kramer, and Levi's results) is that equity returns exhibit a strong seasonal cycle and Treasury returns exhibit a smaller seasonal pattern that is counter-cyclical relative to the seasonal pattern in equity returns. Another interpretation is that equity returns exhibit a strong seasonal cycle whereas the very short end of the Treasury security market does not exhibit a strong cycle (unlike the rest of the Treasury maturity spectrum), perhaps due to the influence of the Federal Reserve. Note that either interpretation is at odds with standard asset pricing models, which imply no seasonal variation in either set of returns. Thus we consider whether we are able to reconcile the stylized fact of seasonal cycles in equity and Treasury returns by developing an asset pricing model that allows for seasonal variation in investors' appetite for risk. For ease of reference, Table III reproduces the 1959-2007 average return values from Table II. It is these return characteristics we attempt to match in our calibration exercise detailed below.

Our next task is to match model-predicted expected returns with observed return patterns. (Note that we occasionally suppress the qualifier "expected" when referring to the expected returns

Table III
Average Realized U.S. Real Returns Over 1959-2007
(6-Month Rates of Return)

Period	VW Equity Return (%)	Risk-free Rate 6-Month Maturity (%)
April-September	1.135	0.939
October-March	6.442	0.938

Note to Table III: This table reports, for ease of reference, the 1959-2007 average realized real equity and risk-free 6-month return data shown in Panel A of Table II. These are the primary return characteristics we attempt to match in our calibration exercise.

on the risky and risk-free assets. All returns reported in the tables that follow are expected returns, produced by solving the system of equations (10) - (13) for the equilibrium values of w_i^k and B_i^k and integrating over growth states.) To do this we must parameterize the consumption growth process of our model and determine the seasonal patterns of expected returns. We explore two alternative calibrations to model consumption growth.

The first is based on consumption data from January 1959 to December 2007. We restrict our attention to data starting in 1959 due to availability of the risk-free 6-month Treasury bill return series and consumption data. We restrict our end date to 2007, as a practical matter because macroeconomic data including consumption data is revised for years after initial release, as well as to avoid the impact of the financial crisis, which caused large movements in both equity and risk-free rates, arguably unrelated to our focus of interest. Enlarging our data to include returns from the crisis period, however, does not materially impact estimates of risk premia. The consumption data we employ for this exercise are real nondurables and services consumption (from the Bureau of Economic Analysis). We find positively autocorrelated consumption growth, and assuming a two-state Markov process for consumption growth, we derive $\phi(i, i) = 0.67$ (compared to Mehra and Prescott's estimate of 0.43). The mean (annualized) real growth rate in consumption we find is 0.0332 together with a standard deviation of 0.0152. The second calibration of consumption growth we utilize is the classic parameterization of Mehra and Prescott (1985). This calibration exhibits negative autocorrelation of consumption growth, mean consumption growth equal to 1.8%, and standard deviation of growth equal to 3.6%. Campbell (1986), Abel (1999), and Lettau, Ludvigson, and Wachter (2008) employ a scaling parameter (denoted λ by Abel (1999) and others). The Lucas model has $\lambda = 1$. Values greater than 1 are used for approximating leveraged assets. We follow Lettau, Ludvigson, and Wachter (2008) and set $\lambda = 4.5$ for both calibrations.

The seasonal rates of return for the risky and risk-free asset are determined using the CRSP value-weighted returns including dividends for the risky asset, and the CRSP Fama Treasury Bill Term Structure Files for the risk-free assets. Because these are nominal rates of return and our model describes real rates of return, we must deflate these return data. We deflate the nominal return series following Mehra and Prescott (1985) using a deflator series produced by dividing real consumption of non-durables and services by the nominal consumption of non-durables and services.[16]

Based on this information, we determine the expected returns to holding risk-free and risky assets implied by this set of information by searching over a grid of values of the risk aversion and IES preference parameters. The grid we search over is informed by previous research. The parameter of relative risk aversion, γ, has been argued to be possibly as high as 30 by Lettau et al. (2008) (to match observed mean equity premia, dividend yield, and risk-free rates on post-war data). Small negative values of the IES parameter (denoted ψ) have been reported by authors including Hall (1988), but for the most part the consensus in the literature is that the parameter is positive and, according to researchers including Lettau et al. (2008) and Vissing-Jorgensen et al. (2003), greater than 1. Vissing-Jorgensen et al. (2003) suggest values close to 1.5 but report values as high as 17.6 (see their Table I).

Since we explore seasonally varying risk aversion and IES in this calibration exercise, each (γ_L, ψ_L) spring/summer pair has a corresponding fall/winter (γ_H, ψ_H) pair with $\gamma_H \neq \gamma_L$ and $\psi_H \neq \psi_L$. (Later, we also consider varying only one of ψ or γ at a time.) We explore values of γ_L as low as 1.5 and as high as 30. We set $\gamma_H = \gamma_L + \Delta\gamma$, with $\Delta\gamma$ equal to as little as 0.25 and as much as 30. We consider values of ψ between 1 and 6.[17] We set $\psi_H = \psi_L - \Delta\psi$, with $\Delta\psi$ equal to as little as 0.0001 and as much as 0.1. In all cases we set δ no greater than 0.9849 for the semi-annual frequency.[18,19]

3.2 Calibration with Seasonally Varying Risk Aversion and IES

We demonstrate now that with seasonally varying risk aversion and IES, we can closely match the observed magnitudes and signs of changes in returns in our semi-annual stock and bond data. We present results in Appendix B showing (i) if we allow only risk aversion to vary seasonally (with

[16] The T-bill series are deflated using the *predicted* inflation rate, where we use an ARMA(1,1) time series model to form our predicted inflation series. We use predicted inflation because the T-bill rate is set using information available only at the beginning of the period; realized inflation is known only after the T-bill matures. This regression, estimated on semi-annual data, has an R-squared of 71% and removes evidence of autocorrelation to 5 lags (2 1/2 years). Coefficient estimates (with standard errors in parentheses) are as follows. Intercept: 0.0029 (0.0013), AR(1): 0.86 (0.058), MA(1): -0.097 (0.120). The *realized* inflation rate is used to deflate the equity return, because the realized inflation rate is known when the equity return is realized.

[17] In a previous version of this paper we considered values of $\psi < 1$ as well. Doing so allows slightly better fit to equity and bond returns for the case of constant ψ but has little or no impact on fit when ψ is allowed to vary seasonally. Here we restrict our attention to values of $\psi > 1$ due to the growing consensus in the literature.

[18] We chose an semi-annual rate of time preference of 0.9849 to match an annual rate of 0.97, based on the quarterly value of 0.9925 from Lettau et al. (2008).

[19] There are (γ, ψ) pairs for which a simultaneous setting of $\delta = 0.9849$ can produce negative returns; when this occurs we lower δ in steps by roughly 0.01 and restart. This is continued until returns are positive. Virtually all of the parameterizations have a δ no less than 0.97.

IES held constant), we are able to replicate the signs of seasonal changes in risky versus risk-free asset returns, but not the magnitudes and (ii) if we allow only IES to vary seasonally (with risk aversion held constant), we are unable to match the signs or the magnitudes of seasonal changes in risky and risk-free assets.

A representative set of the best-performing results is presented in Table IV based on seasonally varying risk aversion and IES and using the Mehra and Prescott consumption parameters. Values for γ and ψ are in the first two columns, with results clustered in groups of two, the first corresponding to the low risk aversion period April-September, and the second the high risk aversion period, October-March. The equity and risk-free rates of return produced by the various γ and ψ combinations are reported, together with the equity volatility and a test that the data sample moments reported in Table III are consistent with the model.[20] Analogous results using the 1959-2007 consumption parameters appear in Table V.[21] We postpone discussion of the equity volatility results in both tables until later. With either set of calibrations, we obtain the stylized seasonal patterns in returns, with markedly higher expected equity returns and slightly lower or very similar magnitude expected risk-free rates in the high risk aversion season relative to the low risk aversion season. Further, we come fairly close to matching the level of returns in both seasons for a range of parameter values.

Specifically, consider the results in Table IV. In the top set of cells, γ varies from 1.5 in the low risk aversion season to 8.5 in the high risk aversion season and ψ varies from 1.5 in the low risk aversion season to 1.494 in the high risk aversion season. The magnitude of equity returns across the seasons, 1.9% and 8.4%, come close to matching the observed values shown in Table III, as do the magnitude of risk-free rates across the seasons, 0.16% and 0.28%. Similarly, throughout the table, the results capture the primary features of the data: the large swing in equity returns (as large as a 7.0% difference across the seasons), the small swing in risk-free returns (as small as a few basis points across the seasons), and the seasonal patterns in risk-free returns typically being offset in timing relative to equity returns. Analysis based on the 1959-2007 consumption

[20]The model specification test has us comparing the model-implied return patterns with the return patterns observed in the actual data, following Gregory and Smith (1991) and Cogley and Nason (1995), among others. The details of this method are as follows. Using the γ and ψ values from the table, the consumption growth parameters reported in the paper, and Equations (10) - (13), we simulate 10,000 independent outcomes of 96 periods each (to match the 48 year semi-annual actual data period to which we calibrate our model). The Markov transition probabilities introduce randomness to these 10,000 simulated outcomes. We then count the fraction of these simulated economies that yield results like those found in U.S. data. That is, we count how many simulated economies had low (high) risk aversion period equity returns that were no higher (lower) than the sample April-September (October-March) equity return of 1.135% (6.442%), and low (high) risk aversion period risk-free returns that were no lower (higher) than the sample April-September (October-March) risk-free return of 0.939% (0.938%).

[21]Note that the Mehra and Prescott data and the 1959-2007 data lead to fairly different values of the mean and variance of consumption growth for calibrating. (The mean growth is lower and the volatility is higher with the Mehra and Prescott sample.) This difference leads to different (γ, ψ) pairs across tables when we match to equity and risk-free returns. That is, to replicate the features of the data, we need different values of γ and ψ depending on whether we calibrate to the features of consumption growth from the Mehra and Prescott sample period or the longer sample period.

Table IV
Calibration Results for Seasonally Varying Risk Aversion and Seasonally Varying IES: Using Mehra and Prescott Consumption Parameters (6-Month Rates of Return)

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
1.5	1.5	April-September	1.9	0.16	1.08	
8.5	1.494	October-March	8.4	0.28	1.23	.20520
2.5	1.5	April-September	3.2	0.31	1.10	
8.5	1.495	October-March	8.5	0.29	1.23	.07600
1.5	1.25	April-September	2.7	0.90	1.31	
10.5	1.247	October-March	9.7	0.20	1.46	1.0000
1.75	3.5	April-September	2.8	0.88	0.47	
10.75	3.45	October-March	9.4	0.45	0.75	1.0000

Note to Table IV: This table summarizes results where we allow both seasonally varying risk aversion and seasonally varying IES, and where we calibrate to consumption parameters from the Mehra and Prescott (1985) sample period: negative autocorrelation of consumption growth ($\phi = 0.43$), mean consumption growth equal to 1.80%, and standard deviation of growth equal to 3.60%.

Table V
Calibration Results for Seasonally Varying Risk Aversion and Seasonally Varying IES: Using the 1959-2007 Consumption Parameters (6-Month Rates of Return)

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
2	3	April-September	0.9	0.33	0.54	
22	2.98	October-March	6.1	0.38	1.16	.09080
1.75	1.75	April-September	1.0	0.56	0.93	
26.75	1.745	October-March	6.0	0.42	1.44	.09120
3	2.25	April-September	1.2	0.40	0.73	
28	2.241	October-March	6.1	0.22	1.39	.06720
1.75	1.75	April-September	1.0	0.56	0.93	
26.75	1.745	October-March	6.0	0.42	1.44	.09120
3	2.25	April-September	1.2	0.40	0.73	
28	2.241	October-March	6.1	0.22	1.39	.06720
2	2.25	April-September	1.1	0.58	0.73	
27	2.241	October-March	6.0	0.23	1.37	.09720

Note to Table V: This table summarizes results where we allow both seasonally varying risk aversion and seasonally varying IES, and where we calibrate to consumption parameters from the 1959-2007 sample period: positive autocorrelation of consumption growth ($\phi = 0.67$), mean consumption growth equal to 3.32%, and standard deviation of growth equal to 1.52%.

parameters, shown in Table V, also captures the primary features of the realized data, with large seasonal differences in risky returns (as large as 5% difference), smaller seasonal differences in risk-free returns, and typically counter-cyclical seasonal variation in risk-free returns relative to risky returns. Finally, none of these models reject the sample return moments from Table III at the 5% level of significance, as shown by the specification test p-value in the last column of the tables. These effects are captured with changes in the coefficient of relative risk aversion across the seasons as small as 6 with the Mehra and Prescott calibration, and as large as 25 with the 1959-2007 calibration. Larger changes in the coefficient of relative risk aversion are required for the 1959-2007 calibration owing to the smaller volatility of consumption over this period relative to the span of data Mehra and Prescott study.

Recall from Section 1 that although specific estimates vary across studies, clinical research broadly suggests roughly 20% to 40% of the North American population suffers from severe enough SAD or winter blues as to experience functional impairment, and the work of Kramer and Weber (2012) suggests that even healthy individuals experience some degree of seasonal depression and seasonally varying risk aversion. With market participants experiencing significant seasonal changes in risk aversion, the change in the coefficient of relative risk aversion understates the magnitude of the change in these individuals by factor of up to roughly two and a half to five. Multiplying the change in the coefficient of relative risk aversion delivered by the Mehra and Prescott calibration by a factor of two to five still leaves us in the approximate range considered by Lettau et al. (2008), though the 1959-2007 calibration would leave us with anomalously large fluctuations in risk aversion.

The intuition for the different behavior of risky versus risk-free returns as a consequence of seasonally varying investor risk aversion and IES emerges directly from the model. When risk aversion rises in isolation, the price of the risky asset should become relatively lower than it would be otherwise, leading to a higher expected future risky return. But when risk aversion rises, the price of the risk-free asset should become *higher* than it would be otherwise, leading to a *lower* expected future risk-free return. When IES drops in isolation, an agent becomes less willing to postpone his/her consumption. This should lead to a drop in the prices of *both* the risky and risk-free assets which in turn should lead to relatively higher expected returns for both. In Table IV and Table V, both risk aversion and IES are changing. Changing the two simultaneously has an unambiguous impact on the risky return since an increase in risk aversion and a drop in IES should both lead to a higher risky return (and we observe this in Table IV and Table V). The impact on the risk-free return, however, depends on whether the influence of the changing risk aversion outweighs the influence of the changing IES. Results suggest that risk-free returns are more greatly influenced by agents' preference to consume in the present than they are by agents' risk aversion, since the risk-free return ends up lower in the high risk aversion / inelastic consumption period. In short, the combined impact of IES and risk aversion is to attenuate the movement of bond returns in periods when equity return movements become magnified. That is, with seasonally varying IES and risk aversion we are able to match the observed patterns of returns, with equity returns having a strong seasonal and bond returns exhibiting a counter-cyclical seasonal pattern of much smaller magnitude.

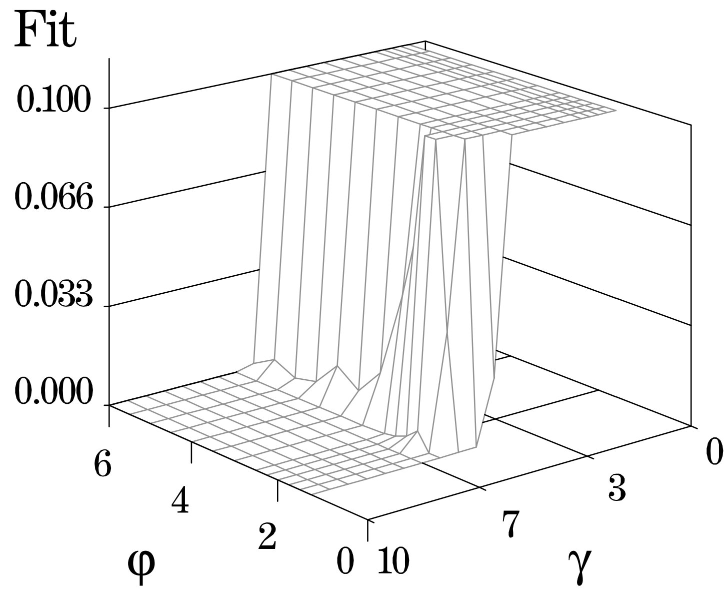

Figure 1: **Best Fit Based on Parameters from the Mehra and Prescott Calibration Period.** Goodness-of-fit measures for the case of seasonally varying IES and seasonally varying risk aversion, based on consumption parameters from the Mehra and Prescott calibration period.

In Figure 1 we plot goodness-of-fit measures (p-values from the specification test described in footnote 20) for the case of seasonally varying IES and seasonally varying risk aversion, using the Mehra and Prescott calibration period. Results for the 1959-2007 calibration period, which are very similar, appear in Figure 2. These figures provide deeper intuition for the values of γ and ψ that yield the best match to observed returns over the 1959-2007 period and provide insight into the sensitivity of the match to the values of γ and ψ. The higher the value on the y-axis (labeled "Fit"), the better the exercise succeeds in matching actual returns. A "Fit" value (or p-value) of zero, the lowest possible value on the scale of the y-axis, would correspond to a very bad fit, with the data rejecting the model very strongly. Terminating the top of the vertical scale at 0.10 is consistent with the fact that we do not typically reject a model based on standard levels of significance when

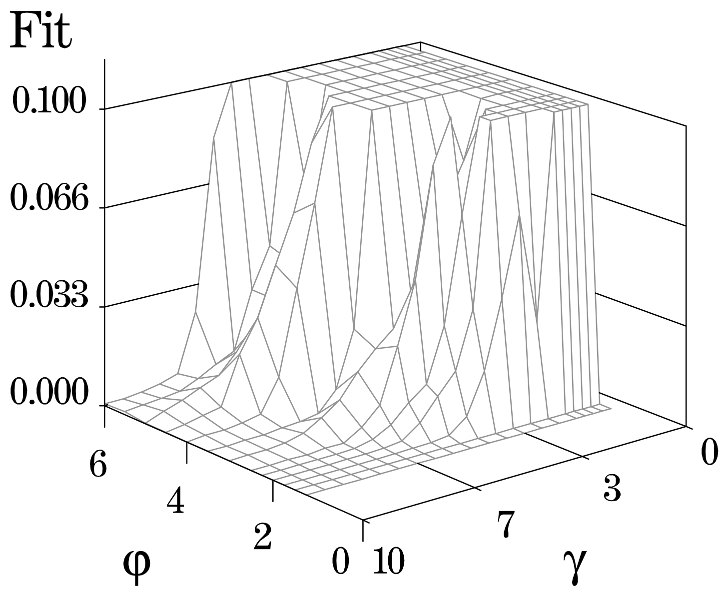

Figure 2: **Best Fit Based on Parameters from the 1959-2007 Calibration Period.** Goodness-of-fit measures for the case of seasonally varying IES and seasonally varying risk aversion, based on consumption parameters from the 1959-2007 calibration period.

we observe p-values of 10% or more. (Results are very similar based on either a 5% or 10% level of significance.) The values plotted on the horizontal and depth axes and are the values of γ and ψ in the low risk aversion period, i.e. γ_L or ψ_L. The best model fit in Figure 1 is achieved for ψ_L values between about 2 and 6, and for γ_L values up to about 7. The best model fit in Figure 2 is similar, though the suitable range for γ_L is more constrained depending on the value of ψ_L.

For comparison, we present two modified versions of this calibration exercise in Appendix B. In one case we consider having seasonally varying IES but constant risk aversion. In the other we consider having seasonally varying risk aversion but constant IES. We find that when we restrict the preference parameters to allow seasonal variation in only one or the other of IES and risk aversion, we are unable to match simultaneously the return magnitudes and the seasonal patterns in risky versus risk-free returns. Specifically, with seasonally varying risk aversion and constant IES, we are able to match the sign of the seasonal changes in the risky versus risk-free assets, however the magnitudes of the changes are a poor match to observed magnitudes. We can get close to observed changes in magnitudes of returns only with extremely large changes in γ, typically in excess of 50. When we allow seasonally varying IES but restrict risk aversion to be constant across the seasons, we have even poorer performance.

4 Implications for Volatility

The model we consider in this paper has clear implications for seasonal cycles in risky and risk-free asset returns, and we have explored these implications extensively above. Indeed, resolving whether such a model can match previously documented return regularities is what motivates this study. An interesting further implication of the model is for risky asset return volatility: we find the volatility of market returns behave pro-cyclically relative to risk aversion, taking on higher values in seasons when the representative agent suffers from seasonal depression, even in absence of variation in consumption risk.

In our model, the volatility of the risky assets' returns *across seasons* comes from two sources, volatility of consumption growth (which we have set to be constant across seasons, but which we allow to vary in Section 5) and variation in risk aversion. For a given season (i.e., for a given level of risk aversion) the volatility of returns comes solely from consumption growth, and the higher the expected return, the larger the impact of consumption growth shocks. This result can be derived from Equation (7), in which the return on the risky asset is expressed in terms of the price-dividend ratio at the end of the season and consumption growth, divided by the price dividend ratio at the beginning of the season. Calculating the volatility of returns based on that expression reveals that the volatility of returns is proportional to the volatility of the price-dividend ratio divided by its lag. The volatility of the price-dividend ratio is itself proportional to the magnitude of the price-dividend ratio, so that Equation (7) implies return volatility is proportional to the level of returns. We also know that in the high risk aversion season, investors must be compensated with higher expected returns in order to hold the risky asset, thus return volatility should also be higher in the high risk aversion season, even though the high risk aversion season has the investor facing the same consumption growth risk as in the low risk aversion season. That is, return volatility should behave pro-cyclically relative to risk aversion in our model. We turn now to testing whether this prediction is supported by the data.

Table VI contains realized U.S. equity return volatility values for the 1959-2007 sample and various sub-periods, based on S&P 500 real return data. The data are strongly consistent with the predicted co-movement of volatility and returns. The season during which many investors experience depression, October-March, has higher returns and higher volatility in the whole sample, in the first and second halves of the sample, and in every decade (except the 1960s for volatility). Kamstra, Kramer, and Levi (2011) demonstrate that seasonality in market risk does not account for the empirical evidence of a seasonal-depression-related cycle in equity and bond returns. Thus a contribution of our model is that seasonality in risk aversion may lead to seasonality in mean returns *and in volatility.* Given that changes in risk aversion can cause changes in the volatility

Table VI
Realized U.S. Equity Return Volatility

Period	1959-2007	1959-1983	1984-2007	1959-1969	1970-1979	1980-1989	1990-1999	2000-2007
April-September	9.61	9.91	9.29	9.01	8.38	11.06	9.57	10.20
October-March	12.20	10.69	13.77	8.94	11.92	14.90	13.75	11.74
Seasonal Change	2.60	0.78	4.48	-0.07	3.54	3.84	4.18	1.53

Note to Table VI: This table contains volatility measures based on realized volatility for the 1959-2007 period and various sub-periods. Monthly nominal equity returns are squared and summed semi-annually, then the square root of this sum is annualized. For reference to the theoretical justification for a realized volatility measure, see Andersen et al. (2003). To calculate the significance of the differences in volatilities, we performed the identical systems equations estimation used in Table II on rates (calculated as described in footnote 15), but now using squared values of the rates. Significance of the tests for seasonal difference in rates are indicated as follows: one, two, and three asterisks represent significance at the 10%, 5% and 1% respectively, based on two-tailed tests.

of asset returns, an implication is that the relationship between volatility and returns may become difficult to disentangle in practice.

With our model calibrations successfully matching the sign of seasonal differences in risky asset returns and risky asset return volatility, as well as the level of risky asset returns, a natural question is whether we are also able to match the *seasonal change* in volatility, where volatility over 1959-2007 is roughly 150 basis points higher in the high risk aversion season (October-March), or roughly 15% higher, proportionately. Note that we do not expect to match the *level* of return volatility with the simple asset pricing model we employ here.[22]

Consider the second-to-last column of Table IV and Table V, where we present the volatility figures based on the 1959-2007 calibration and the Mehra and Prescott calibration respectively. In both tables, we see that the model yields volatility which is roughly 20 to 60 basis points higher in the high risk aversion season, or an increase of 20 to 100% proportionately. Thus, although our model is unable to match the level of the return volatility, it is able to closely match the *proportional change* in volatility observed across the two seasons.

5 Consumption Seasonality

Now we consider whether the seasonal patterns we observe in equity and Treasury returns arise simply as a consequence of deterministically and seasonally varying consumption growth. It is established that consumption seasonality can dramatically impact asset returns in consumption-based asset pricing models. See for instance, Miron (1986) and Ferson and Harvey (1992). Piazzesi (2001) documents and discusses seasonal cross-correlations between consumption growth and asset

[22]See Bansal and Yaron (2004) for a model capable of matching return volatility. Bansal and Yaron exploit an Epstein and Zin (1989) representative agent model with recursive preferences, as we do, and augment this model with seasonally varying consumption volatility and a consumption growth process that incorporates a small, seasonally varying, but persistent and predictable component.

returns in the context of Gabaix and Laibson's (2001) paper on resolving the equity premium puzzle, seasonalities which appear in spite of working with seasonally adjusted consumption data. Piazzesi contends that this seasonality may matter for stock pricing, implying that consumption growth predictability may need to be endogenized in asset pricing models. Jagannathan and Wang (2007) find that the consumption CAPM is much better able to explain the cross-section of returns when annual growth is measured from fourth quarter to fourth quarter. They point out that investors are likely to revise both consumption and investment decisions simultaneously in the fourth quarter (for instance, due to the timing of the tax year, for reasons related to culture, or because of the timing of important events such as year-end bonuses) and that there is seasonality in the degree of covariance between consumption growth and asset returns.

Once again, following Mehra and Prescott, we assume a two-state Markov world, but we now allow the consumption growth rate to have a different mean and variance in the high risk aversion season compared to the low risk aversion season, as is evident in the data. The Markov transition matrix remains the same, $\phi(i,i) = 0.67$ (based on the 1959-2007 calibration period), but now the growth rate depends on both the season and the state. In our data, consumption has a lower growth rate and lower variance in the fall/winter season than in the spring/summer season, and we replicate this feature of the data in our calibration exercise. Later we report specific values of the growth rate by season and by state.

Let us define the consumption growth mean and standard deviation values in the high risk aversion season as $\{\mu_H, \sigma_H\}$ and the consumption growth mean and standard deviation values in the low risk aversion season as $\{\mu_L, \sigma_L\}$. Further, the consumption growth rate realized in the high risk aversion season is g_H and the consumption growth rate realized in the low risk aversion season is g_L. Modifying Equation (6) to incorporate the state-dependent consumption growth rates then yields the following system of equations for price-dividend ratios,

$$(w_i^H)^{\theta_H} = \delta^{\theta_H} \sum_{j=1}^{2} \phi(i,j) g_{j,H}^{1-\gamma_H} (1 + w_j^L)^{\frac{\theta_L(1-\gamma_H)}{(1-\gamma_L)}} \tag{14}$$

$$(w_i^L)^{\theta_L} = \delta^{\theta_L} \sum_{j=1}^{2} \phi(i,j) g_{j,L}^{1-\gamma_L} (1 + w_j^H)^{\frac{\theta_H(1-\gamma_L)}{(1-\gamma_H)}}. \tag{15}$$

As before, $i = 1, 2$ indexes states, $k = H, L$ indexes different sets of preference parameters (in the high and low risk aversion seasons), and ϕ is the Markov state-transition probability matrix. Here w_i^k has the interpretation of price-dividend ratios when the preference parameters are γ_k and θ_k and the current state of consumption growth is i.

In the case of a two-state Markov world and two sets of preference parameters, it follows from

Table VII
Annualized Average Consumption Growth Rates and Volatility

Period	Mean Consumption Growth (μ)	Volatility of Consumption Growth (σ)
Panel A: 1959-2004		
April-September	4.0%	3.1%
October-March	2.4%	1.3%
Panel B: 1959-1983		
April-September	4.3%	4.0%
October-March	2.4%	1.6%
Panel C: 1984-2004		
April-September	3.8%	1.7%
October-March	2.4%	0.8%

Note to Table VII: We report estimates of the mean consumption growth rate and the volatility of consumption growth for the April-September and October-March seasons over the 1959-2004 sample period and sub-periods consisting of the first and last halves of the sample. The source of the consumption data is described in footnote 23, and its deflation is described in footnote 24.

Equation (9) that the one-period bond price satisfies the following system of equations:

$$B_i^H = \delta^{\theta_H}(w_i^H)^{1-\theta_H}\sum_{j=1}^{2}\phi(i,j)g_{j,H}^{-\gamma_H}(1+w_j^L)^{\frac{\theta_L(1-\gamma_H)}{(1-\gamma_L)}-1}, \qquad (16)$$

$$B_i^L = \delta^{\theta_L}(w_i^L)^{1-\theta_L}\sum_{j=1}^{2}\phi(i,j)g_{j,L}^{-\gamma_L}(1+w_j^H)^{\frac{\theta_H(1-\gamma_L)}{(1-\gamma_H)}-1}. \qquad (17)$$

In Table VII, we provide average annual consumption growth rates estimated over 1959-2004 as well as sub-periods covering the first and last halves of the sample.[23,24] We find that the estimates (especially the means) are very stable across the full sample and the sub-periods, and the seasonality across periods is very robust. Specifically, the consumption growth rate and its volatility are always lower in the high risk aversion season.

We proceed now to the calibration exercise based on the adapted Epstein and Zin model. Let us consider first the case with time-invariant preferences, $\{\gamma_H, \theta_H\} = \{\gamma_L, \theta_L\}$. Searching over a wide range of parameter values (identical to those described in Section 3 above) we find without exception that the model delivers lower equity returns in the October-March season than in the April-September season, which is at odds with the observed return pattern over these seasons. The model also delivers, virtually without exception, lower bond returns in the April-September

[23] Our seasonally unadjusted consumption data is the sum of non-durables and services: Variables B004RU1 and B005RU1 from Table 8.1, Bureau of Economic Analysis, 1947:1-2004:4. (BEA has not updated this series recently due to budget cutbacks.)

[24] We deflate consumption with the consumer price index for all urban consumers, all items, series ID CPIAUCNS, U.S. Department of Labor: Bureau of Labor Statistics, using the CPI at the end of the quarter used as the deflator.

Table VIII
Calibration Results Incorporating Seasonal Consumption Growth: Using 1959-2007 Consumption Parameters (6-Month Rates of Return)

Panel A: Constant γ & ψ

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
1.5	4.5	April-September	2.8	0.85	0.74	
1.5	4.5	October-March	2.1	1.39	0.31	.00000
2.25	2.25	April-September	3.3	0.59	1.52	
2.25	2.25	October-March	2.4	1.47	0.64	.00000
2.5	1.75	April-September	3.4	0.54	1.95	
2.5	1.75	October-March	2.5	1.57	0.82	.00000
3.5	1.5	April-September	4.2	0.31	2.32	
3.5	1.5	October-March	3.2	1.95	0.97	.00000

Panel B: Seasonally Varying γ & ψ

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
1.5	3	April-September	2.0	0.53	1.11	
26.5	2.98	October-March	7.3	0.62	1.42	.05890
1.5	3	April-September	2.6	1.19	1.12	
41.5	2.99	October-March	7.2	0.42	1.71	.04260
1.5	5.5	April-September	2.4	0.75	0.60	
21.5	5.45	October-March	7.0	0.06	1.21	.00470
1.5	4.5	April-September	2.0	0.38	0.74	
21.5	4.45	October-March	7.3	0.48	1.23	.13200

Note to Table VIII: We report results where we allow seasonally varying consumption, using consumption parameters from the 1959-2004 sample period: positive autocorrelation of annual consumption growth ($\phi = 0.67$), and seasonal consumption growth parameters of $\{\mu_H, \sigma_H\}$ ={2.4%,1.3%} and $\{\mu_L, \sigma_L\}$ ={4.0%,3.1%}.

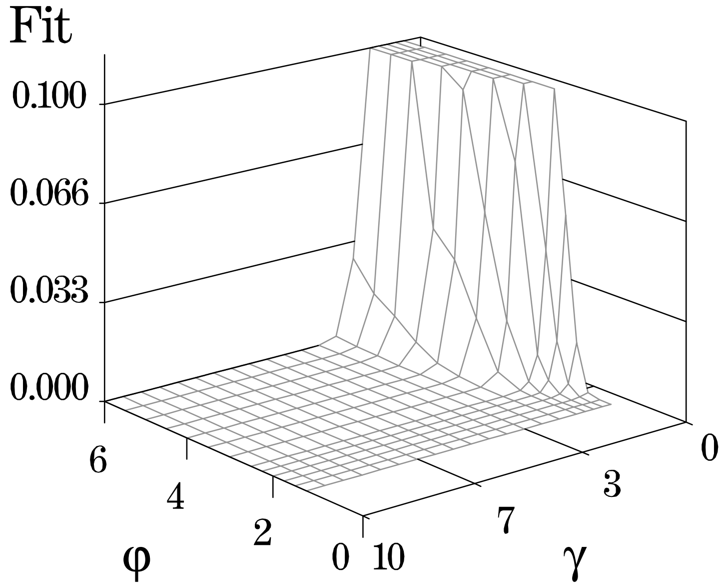

Figure 3: **Best Fit for Seasonally Varying Consumption Growth and Risk Case, Based on Parameters from the 1959-2007 Calibration Period.** Goodness-of-fit measures for the case with seasonally varying consumption growth and risk, and seasonally varying IES and risk aversion, based on consumption parameters from the 1959-2007 calibration period.

season. The top half of Table VIII contains a representative set of the best-performing results for the time-invariant preference calibration exercise. In sum, the exercise yields results that are strongly inconsistent with observed data, leading us to conclude that seasonality in consumption growth does not explain the observed seasonal patterns in returns.

We turn next to allowing for both seasonality in consumption growth *and* seasonality in preferences. That is, we allow both γ and θ to change with the semi-annual seasons, as in Section 2.2. We also allow for consumption growth rates to have a lower mean and a lower variance in the high risk aversion season, matched with observed data. In this calibration exercise, we find we are able to match the direction and, to a remarkable extent, the magnitude of changes in risky and risk-free returns. Representative results are presented in the bottom half of Table VIII.

In Figure 3 we plot the goodness-of-fit measure for the case of seasonally varying IES, seasonally varying risk aversion, and seasonally varying consumption growth and risk.[25] The best fit is achieved for γ values up to about 2 or 3 and for ψ values greater than about 3.

To sum up, we find that when allowing for seasonality in consumption growth alone we are

[25]The goodness-of-fit measures for the level and for the change of returns are calculated as described in footnote 20.

unable to explain the observed seasonal patterns in risky and risk-free returns. To capture the seasonal variability of returns, we must also allow for seasonality in preferences, specifically both seasonally varying risk aversion and seasonally varying IES.

6 Conclusions

Any model of asset returns used to bridge the empirical-conceptual gap must ultimately be based on realistic preferences for consumption over time or preferences for risk. We explore the ability of conventional asset pricing models to match a remarkable empirical regularity, namely that realized equity returns vary conditionally across the seasons as much as 12% (annualized) and that average U.S. Treasury returns vary counter-cyclically relative to equity returns, albeit on a much smaller scale.

We find that a model incorporating seasonality in the willingness to bear risk (while holding constant the willingness to delay or accelerate consumption, the intertemporal elasticity of substitution, across the seasons) matches the observed opposing signs of seasonal changes in risky versus safe asset returns but is unable to closely match the magnitude of the seasonal changes in returns. We also find that allowing seasonal variation in the IES (while holding risk aversion constant across the seasons) is insufficient to match even the stylized patterns of offset seasonal cycles in risky versus safe asset returns. Finally, we find that the observed characteristics of risky and risk-free returns can be well matched if we allow annual cycles in both risk aversion and IES. Specifically, we find expected equity returns are higher and expected bond returns are lower in the fall/winter season (the time of year when some investors experience increased depression and risk aversion), and we find the variation in bond returns across the seasons is much smaller than that in equity returns.

An interesting and unexpected result is that time variation in risk aversion and IES can lead to systematic changes in return volatility (with no variation in consumption risk) and this matches the regularity in the data coincident with the timing of seasonal depression and seasonal risk aversion: risky asset volatility is greater in the high risk aversion season. Finally, we consider a strong alternative to seasonally varying preference for risk: seasonally varying consumption growth and consumption risk. We find seasonality in consumption growth is unable to produce the higher equity returns that are observed in the fall/winter season, nor is it able to produce counter-cyclically varying risk-free returns.

Relative to prior work on the influence of seasonally varying preferences in asset pricing, our findings with respect to IES are novel. Specifically, we find that seasonal changes in agents' willingness to postpone consumption may play an important and previously overlooked supportive role in explaining seasonal return dynamics.

On the empirical front, there is extensive evidence of significant correlation between seasonal depression and a range of financial quantities, including Treasury returns, stock returns, mutual fund flows, bid-ask spreads, initial public offering returns, real estate investment trusts, and analysts' earnings estimates (see footnote 2). In contrast, there has been relatively little exploration of the theoretical underpinnings of these relationships, in particular whether reasonable parameterizations of standard models could possibly generate the observed consequences in financial markets. This paper represents an attempt to bring our understanding of the theory closer to what we observe empirically. The convergence of theoretical implications of a state representative agent model with reasonable parameterization to empirical observations should provide a greater degree of confidence in the relevance of seasonal depression in particular and time-varying preferences in general for the study of financial markets.

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Appendices

A Recursive Utility with Seasonal Depression

Let us consider recursive utility with changing risk aversion γ_t and constant ϕ:

$$U_t = \left[c_t^{(1-\gamma_t)/\theta_t} + \delta(E_t U_{t+1}^{1-\gamma_t})^{1/\theta_t}\right]^{\theta_t/(1-\gamma_t)}$$

where ψ, defined through $\theta_t = (1-\gamma_t)/(1-1/\psi)$, is the elasticity of intertemporal substitution. When $\theta_t = 1$ we get the standard intertemporally additive expected utility.

The utility maximization problem is

$$J_t(W_t, x_t) = \max_{c_t, \pi_t} \left[c_t^{(1-\gamma_t)/\theta_t} + \delta(E_t J_{t+1}^{1-\gamma_t})^{1/\theta_t}\right]^{\theta_t/(1-\gamma_t)}$$

subject to the constraint

$$W_{t+1} = (W_t - c_t)\pi_t(1 + R_{t+1})$$

where x_t is the vector of state variables and π_t is the vector of portfolio weights.

It is readily seen that the recursive utility is homothetic so that $J_t(W_t, x_t) = f_t(x_t)W_t$. The original utility maximization problem can be written as

$$\begin{aligned} f_t(x_t)W_t &= \max_{c_t,\pi_t} \left[c_t^{(1-\gamma_t)/\theta_t} + \delta\left(E_t\left[(f_{t+1}(x_{t+1})(W_t - c_t)\pi_t(1+R_{t+1}))^{1-\gamma_t}\right]\right)^{1/\theta_t}\right]^{\theta_t/(1-\gamma_t)} \\ &= \max_{c_t,\pi_t} \left[c_t^{(1-\gamma_t)/\theta_t} + \delta(W_t - c_t)^{(1-\gamma_t)/\theta_t}\left(E_t\left[(f_{t+1}(x_{t+1})\pi_t(1+R_{t+1}))^{1-\gamma_t}\right]\right)^{1/\theta_t}\right]^{\theta_t/(1-\gamma_t)}. \end{aligned}$$

The first order condition for c_t is then

$$c_t^{(1-\gamma_t)/\theta_t - 1} = \delta(W_t - c_t)^{(1-\gamma_t)/\theta_t - 1}\left(E_t\left[(f_{t+1}(x_{t+1})\pi_t(1+R_{t+1}))^{1-\gamma_t}\right]\right)^{1/\theta_t}.$$

Homogeneity suggests that $c_t = C_t(x_t)W_t$. Substitution into the utility maximization problem yields

$$f_t(x_t)^{(1-\gamma_t)/\theta_t} = C_t(x_t)^{(1-\gamma_t)/\theta_t} + \delta(1 - C_t(x_t))^{(1-\gamma_t)/\theta_t}\left(E_t\left[(f_{t+1}(x_{t+1})\pi_t(1+R_{t+1}))^{1-\gamma_t}\right]\right)^{1/\theta_t}.$$

The first order condition for c_t can be written as

$$C_t(x_t)^{(1-\gamma_t)/\theta_t - 1} = \delta(1 - C_t(x_t))^{(1-\gamma_t)/\theta_t - 1}\left(E_t\left[(f_{t+1}(x_{t+1})\pi_t(1+R_{t+1}))^{1-\gamma_t}\right]\right)^{1/\theta_t}.$$

Combining the two equations yields

$$\begin{aligned} f_t(x_t) &= C_t(x_t)^{(1-\gamma_t-\theta_t)/(1-\gamma_t)}, \\ f_{t+1}(x_{t+1}) &= C_{t+1}(x_{t+1})^{(1-\gamma_{t+1}-\theta_{t+1})/(1-\gamma_{t+1})}, \end{aligned}$$

and

$$C_t(x_t)^{1-\gamma_t-\theta_t} \quad = \quad \delta_t^{\theta}(1-C_t(x_t))^{1-\gamma_t-\theta_t}E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{1-\gamma_t}\right].$$

For the optimal portfolio choice, we examine

$$\max_{\pi_t} E_t\left[(f_{t+1}(x_{t+1})\pi_t(1+R_{t+1}))^{1-\gamma_t}\right].$$

The first order condition is simply

$$0 \quad = \quad E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{-\gamma_t}(R_{i,t+1}-r_t)\right]$$

which implies

$$E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{-\gamma_t}(1+R_{i,t+1})\right] = E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{-\gamma_t}(1+r_t)\right]$$

and

$$E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{1-\gamma_t}\right] = E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{-\gamma_t}(1+r_t)\right].$$

Then

$$C_t(x_t)^{1-\gamma_t-\theta_t} \quad = \quad \delta_t^{\theta}(1-C_t(x_t))^{1-\gamma_t-\theta_t}E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{-\gamma_t}(1+r_t)\right]$$

and

$$C_t(x_t)^{1-\gamma_t-\theta_t} \quad = \quad \delta_t^{\theta}(1-C_t(x_t))^{1-\gamma_t-\theta_t}E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(\pi_t(1+R_{t+1}))^{-\gamma_t}(1+R_{i,t+1})\right].$$

If there is only one risky stock, then

$$C_t(x_t)^{1-\gamma_t-\theta_t} \quad = \quad \delta_t^{\theta}(1-C_t(x_t))^{1-\gamma_t-\theta_t}E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}(1+R_{t+1})^{1-\gamma_t}\right].$$

A.1 The Market Portfolio

In equilibrium, the representative agent holds the market portfolio. The equation derived above can be written as

$$1 \quad = \quad \delta_t^{\theta}\left(\frac{1}{C_t(x_t)}-1\right)^{1-\gamma_t-\theta_t} E_t\left[f_{t+1}(x_{t+1})^{1-\gamma_t}\left(\frac{P_{t+1}+d_{t+1}}{P_t}\right)^{1-\gamma_t}\right]$$

where P_t is the price of the market portfolio. Homogeneity suggests that $P_t = w(x_t)d_t$. Now note that in equilibrium $c_t = d_t$ and consumer wealth is equal to the stock price because there is only one share of the stock and the consumer does not have labor income. Thus, $W_t = P_t + d_t$ and $C_t(x_t) = c_t/W_t = 1/(w_t(x_t)+1)$. Substituting into the above equation yields

$$1 \quad = \quad \delta^{\theta_t}\left(\frac{P_t+d_t}{d_t}-1\right)^{1-\gamma_t-\theta_t} E_t\left[\left(\left(\frac{d_{t+1}}{P_{t+1}+d_{t+1}}\right)^{\frac{(1-\gamma_{t+1}-\theta_{t+1})}{(1-\gamma_{t+1})}}\right)^{(1-\gamma_t)}\left(\frac{P_{t+1}+d_{t+1}}{P_t}\right)^{1-\gamma_t}\right]$$

where we have used the expression for f_{t+1} derived earlier. Then

$$1 \quad = \quad \delta^{\theta_t} w_t(x_t)^{1-\gamma_t-\theta_t} E_t\left[(w_{t+1}(x_{t+1})+1)^{\frac{(\gamma_{t+1}+\theta_{t+1}-1)(1-\gamma_t)}{(1-\gamma_{t+1})}}\left(\frac{(w_{t+1}(x_{t+1})+1)g_{t+1}}{w_t(x_t)}\right)^{1-\gamma_t}\right].$$

Simplifying yields

$$w_t(x_t)^{\theta_t} = \delta^{\theta_t} E_t\left[g_{t+1}^{1-\gamma_t}(1+w_{t+1}(x_{t+1}))^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}}\right]. \tag{18}$$

Let us assume a finite-state Markov world and that $P(i,d) = w_{it}d$. Assuming there are only two states, Equation (6) yields the following system of equations for price-dividend ratios, w_{ik}, where i indexes for states and k indexes for different levels of risk aversion:

$$(w_i^1)^{\theta_1} \quad = \quad \delta^{\theta_1}\sum_{j=1}^{n}\phi(i,j)g_j^{1-\gamma_1}(1+w_j^2)^{\frac{\theta_2(1-\gamma_1)}{(1-\gamma_2)}} \tag{19}$$

$$(w_i^2)^{\theta_2} \quad = \quad \delta^{\theta_2}\sum_{j=1}^{n}\phi(i,j)g_j^{1-\gamma_2}(1+w_j^1)^{\frac{\theta_1(1-\gamma_2)}{(1-\gamma_1)}}. \tag{20}$$

A.2 The Bond Price

More generally, the price of any stock should satisfy

$$1 \quad = \quad \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[(w_{t+1}(x_{t+1})+1)^{\frac{(\gamma_{t+1}+\theta_{t+1}-1)(1-\gamma_t)}{(1-\gamma_{t+1})}}((w_{t+1}(x_{t+1})+1)g_{t+1})^{-\gamma_t}\left(\frac{P_{j,t+1}+d_{j,t+1}}{P_{j,t}}\right)\right].$$

Thus

$$P_{j,t} \quad = \quad \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[g_{t+1}^{-\gamma_t}(w_{t+1}(x_{t+1})+1)^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}-1}(P_{j,t+1}+d_{j,t+1})\right]. \tag{21}$$

It follows from Equation (21) that the one period bond price satisfies

$$B_t \quad = \quad \delta^{\theta_t} w_t(x_t)^{1-\theta_t} E_t\left[g_{t+1}^{-\gamma_t}(w_{t+1}(x_{t+1})+1)^{\frac{\theta_{t+1}(1-\gamma_t)}{(1-\gamma_{t+1})}-1}\right]. \tag{22}$$

Let us assume a two-state Markov world and that there are only two levels of risk aversion. Then Equation (22) yields the following system of equations for states $i = 1, 2$:

$$B_i^1 = \delta^{\theta_1}(w_i^1)^{1-\theta_1}\sum_{j=1}^{n}\phi(i,j)g_j^{-\gamma_1}(1+w_j^2)^{\frac{\theta_2(1-\gamma_1)}{(1-\gamma_2)}-1}, \quad \text{for odd time periods, } s = 1, 3, \ldots \quad (23)$$

$$B_i^2 = \delta^{\theta_2}(w_i^2)^{1-\theta_2}\sum_{j=1}^{n}\phi(i,j)g_j^{-\gamma_2}(1+w_j^1)^{\frac{\theta_1(1-\gamma_2)}{(1-\gamma_1)}-1}, \quad \text{for even time periods, } u = 2, 4, \ldots \quad (24)$$

Having solved for w_{ik} from Equations (19) and (20), we can now solve for the bond price and hence the risk-free rate. It is seen from these expressions that unlike in the case of expected CRRA utility, risk-free rates are stationary when the representative agent has Epstein-Zin utility.

It should be noted that even if $\theta_t = 1$, that is when elasticity of intertemporal substitution is equal to risk aversion, Epstein-Zin utility with seasonal depression does not reduce to expected utility with seasonal depression, unlike the case without seasonal depression.

B Calibrations with Seasonally Varying IES / Constant Risk Aversion, and Seasonally Varying Risk Aversion / Constant IES

To supplement our primary analysis in which both IES and risk aversion vary by season, we now consider two additional cases. In the first, risk aversion varies seasonally and IES remains constant across the seasons. In the second, IES varies seasonally and risk aversion remains constant across the seasons. We find neither of these supplemental calibrations is able to match the features of observed risky and safe asset returns.

B.1 Calibration with Seasonally Varying Risk Aversion and Constant IES

Table B.I contains a representative set of the best-performing results based on Mehra and Prescott consumption parameters and Table B.II contains results based on the 1959-2007 consumption parameters. Since we explore seasonally varying risk aversion and constant IES in this calibration exercise, each (γ_L, ψ_L) spring/summer pair has a corresponding fall/winter (γ_H, ψ_H) pair with $\gamma_H = \gamma_L + \Delta\gamma$ and $\psi_H = \psi_L$. (Below, we also consider seasonally varying ψ and constant γ.)

With either the 1959-2007 calibration or the Mehra and Prescott calibration, we obtain the stylized seasonal patterns in returns, with higher expected equity returns and lower expected risk-free rates in the high risk aversion season relative to the low risk aversion season. However, this is only achieved with large changes in the coefficient of relative risk aversion or with a startlingly high equity premium, above 8% semiannually or 16% annually. Thus, permitting seasonal variation in risk aversion alone does not facilitate a good match with observed returns data.

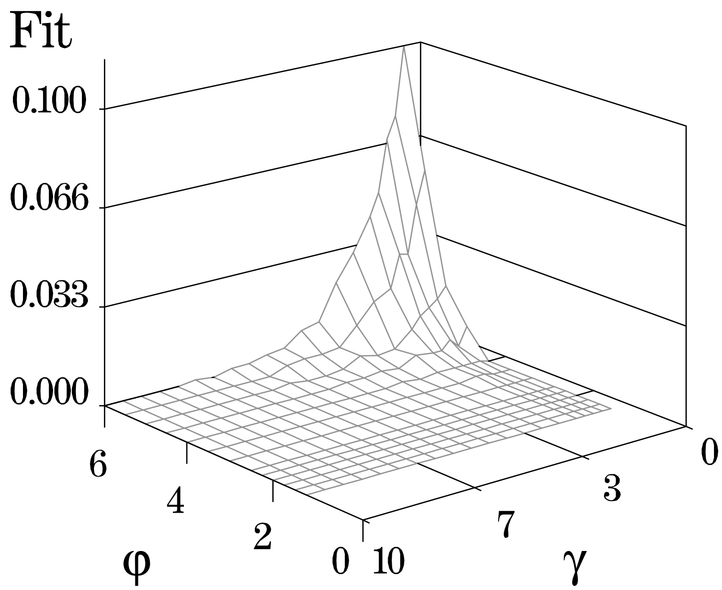

Figure B.1: **Best Fit Based on Parameters from the 1959-2007 Calibration Period.** Goodness-of-fit measures for the case of seasonally varying risk aversion and constant IES, based on consumption parameters from the 1959-2007 calibration period.

In Figure B.1 we plot goodness-of-fit measures as a function of γ_L (the parameterization from the low risk aversion season, spring/summer) and ψ based on the 1959-2007 calibration period. Only very poor fits are available for the 1959-2007 calibration.

There are two issues that require further exploration. First, if we allow IES instead of the coefficient of relative risk aversion to vary across seasons, are we better able to match the magnitude of return changes observed in the data (that is, large equity returns and equity return changes versus very small changes in the risk-free rate)? Second, if so, can we continue to produce offset seasonal cycles in risky versus risk-free returns?

B.2 Calibration with Seasonally Varying IES and Constant Risk Aversion

We now consider the case of seasonally varying IES and constant risk aversion, that is each (γ_L, ψ_L) spring/summer pair has a corresponding fall/winter (γ_H, ψ_H) pair with $\gamma_H = \gamma_L$ and $\psi_H = \psi_L - \Delta\psi$. As before, we search over a very large grid. We provide a representative set of the best-performing results based on the Mehra and Prescott consumption parameter values in Table B.III and based on the 1959-2007 values in Table B.IV. We see that when ψ drops, the expected returns rise for both the risky asset and the risk-free asset. That is, the prices of both risky and risk-free assets drop

Table B.I
Calibration Results for Seasonally Varying Risk Aversion & Constant IES: Using Mehra and Prescott Consumption Parameters (6-Month Rates of Return)

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Model Test P-Value
4	5.5	April-September	6.4	1.69	0.35	
29	5.5	October-March	11.5	0.17	1.14	.09660
1.5	3	April-September	5.8	3.84	0.56	
61.5	3	October-March	11.8	0.22	1.45	1.0000
1.5	1.75	April-September	2.9	1.20	0.93	
2.5	1.75	October-March	3.1	0.36	0.94	.00000
1.5	6	April-September	5.7	3.94	0.28	
16.5	6	October-March	10.6	0.15	0.82	.00010
2.25	4.5	April-September	6.7	3.94	0.38	
22.25	4.5	October-March	11.8	0.54	1.05	.06460

Note to Table B.I: This table summarizes results where we allow seasonally varying risk aversion but do not allow seasonally varying IES, and where we calibrate to consumption parameters from the Mehra and Prescott (1985) sample period: negative autocorrelation of consumption growth ($\phi = 0.43$), mean consumption growth equal to 1.80%, and standard deviation of growth equal to 3.60%.

Table B.II
Calibration Results for Seasonally Varying Risk Aversion & Constant IES: Using the 1959-2007 Consumption Parameters (6-Month Rates of Return)

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
2.5	6	April-September	2.6	1.98	0.28	
62.5	6	October-March	7.1	0.49	1.75	.00560
3	5.5	April-September	2.7	1.93	0.31	
63	5.5	October-March	7.1	0.47	1.75	.00220
8	2	April-September	4.2	1.68	0.91	
15	2	October-March	4.9	0.51	1.10	.00000

Note to Table B.II: This table summarizes results where we allow seasonally varying risk aversion but do not allow seasonally varying IES, and where we calibrate to consumption parameters from the 1959-2007 sample period: positive autocorrelation of consumption growth ($\phi = 0.67$), mean consumption growth equal to 3.32%, and standard deviation of growth equal to 1.52%.

Table B.III
Calibration Results for Seasonally Varying IES and Constant Risk Aversion: Using Mehra and Prescott Consumption Parameters
(6-Month Rates of Return)

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
1.5	4	April-September	2.3	0.70	0.40	
1.5	3.999	October-March	2.4	0.78	0.40	.00000
2	3	April-September	2.5	0.37	0.54	
2	2.9999	October-March	2.5	0.38	0.54	.00000
2	1.25	April-September	2.7	0.36	1.31	
2	1.247	October-March	9.0	6.49	1.39	.00000
2.5	1.5	April-September	3.0	0.23	1.09	
2.5	1.4999	October-March	3.1	0.33	1.10	.00000

Note to Table B.III: This table summarizes results where we do not allow seasonally varying risk aversion but do allow seasonally varying IES, and where we calibrate to consumption parameters from the Mehra and Prescott (1985) sample period: negative autocorrelation of consumption growth ($\phi = 0.43$), mean consumption growth equal to 1.80%, and standard deviation of growth equal to 3.60%.

lower than they would otherwise be in order for all assets to be held in equilibrium, and with the relatively lower prices come relatively higher expected returns for both asset classes. Overall, equity and risk-free returns fail to demonstrate the empirically observed opposing directional movements across the seasons and fail to match the magnitude of returns as well, with risk-free returns varying across the seasons as much as risky returns in many cases.

It is evident from Table B.III and Table B.IV that relatively small changes in ψ can be capable of generating fairly large seasonal changes in return magnitudes in this context, which leads us to several observations. First, note that ψ enters the utility function in a non-linear fashion such that changes in ψ have a diminishing impact on utility as ψ increases. Second, the quantitative impact on returns that arises from a given change in ψ depends on the level of γ. When γ is greater than roughly 2 or 3, a given change in ψ produces a somewhat smaller change in returns than when γ is smaller. These first two points are most clearly evident in Table B.III when ψ is 1.25 and γ is 2; the change in equity and risk-free returns is over 6%. Third, the quantitative impact on returns that arises from a given change in ψ depends on the starting level of ψ. As the starting level of ψ increases, the impact of a given change in ψ on returns becomes smaller.[26]

[26] This is perhaps not surprising: a given change in ψ represents a smaller proportional change (relative to its starting value) the higher the starting value of ψ, and furthermore, ψ impacts utility less markedly as the level of ψ increases.

Table B.IV
Calibration Results for Seasonally Varying IES and Constant Risk Aversion: Using the 1959-2007 Consumption Parameters (6-Month Rates of Return)

γ	ψ	Period	Equity Return (%)	Risk-free Rate (%)	Equity Volatility (%)	Specification Test P-Value
4	3	April-September	2.7	1.12	0.57	
4	2.9999	October-March	2.7	1.13	0.57	.00000
7	1.25	April-September	3.0	0.90	1.36	
7	1.2499	October-March	3.3	1.18	1.36	.00000
5	2	April-September	2.9	1.10	0.86	
5	1.9999	October-March	2.9	1.15	0.86	.00000
1.5	3.5	April-September	2.1	1.54	0.47	
1.5	3.498	October-March	2.4	1.78	0.47	.00000
3	5.5	April-September	2.4	1.13	0.31	
3	5.4999	October-March	2.4	1.13	0.31	.00000

Note to Table B.IV: This table summarizes results where we do not allow seasonally varying risk aversion but do allow seasonally varying IES, and where we calibrate to consumption parameters from the 1959-2007 sample period: positive autocorrelation of consumption growth ($\phi = 0.67$), mean consumption growth equal to 3.32%, and standard deviation of growth equal to 1.52%.