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# VOLATILITY FORECASTS, TRADING VOLUME, AND THE ARCH VERSUS OPTION-IMPLIED VOLATILITY TRADE-OFF

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## Abstract

We investigate empirically the role of trading volume (1) in predicting the relative informativeness of volatility forecasts produced by autoregressive conditional heteroskedasticity (ARCH) models versus the volatility forecasts derived from option prices, and (2) in improving volatility forecasts produced by ARCH and option models and combinations of models. Daily and monthly data are explored. We find that if trading volume was low during period $t-1$ relative to the recent past, ARCH is at least as important as options for forecasting future stock market volatility. Conversely, if volume was high during period $t-1$ relative to the recent past, option-implied volatility is much more important than ARCH for forecasting future volatility. Considering relative trading volume as a proxy for changes in the set of information available to investors, our findings reveal an important switching role for trading volume between a volatility forecast that reflects relatively stale information (the historical ARCH estimate) and the option-implied forward-looking estimate.

*JEL Classification*: G0

## I. Introduction

Market expectations of future return volatility play a crucial role in finance, as does our understanding of the process by which information is incorporated in security prices through the trading process. In this article we examine both of these issues by investigating empirically the role of trading volume (1) in predicting the relative informativeness of volatility forecasts produced by autoregressive

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conditional heteroskedasticity (ARCH) models versus the volatility forecasts derived from option prices, and (2) in improving volatility forecasts produced by ARCH and option models, and combinations of models.

Previous studies report that trading volume does not linearly Granger-cause return volatility but may nonlinearly Granger-cause return volatility (e.g., see Brooks 1998; Heimstra and Jones 1994). The form of the nonlinear relationship between volume and volatility is, however, ambiguous. We provide a simple model with predictive power for forecasting return volatility, where volume plays the role of a switching variable between states in which option-implied volatility is more or less informative than ARCH for volatility forecasting. In particular, when we interact lagged volume with option-implied volatility in an augmented ARCH model, we uncover a significant role for lagged trading volume in forecasting future return volatility. This finding is made possible because of the novel way we incorporate trading volume into our functional forms and because, unlike previous studies that add either trading volume or option-implied volatility (but not both) to ARCH models, we consider all three factors together.

Previous studies that add option-implied volatility to ARCH do so primarily to investigate the efficiency of option markets, not to improve ARCH forecasts per se. If option markets are efficient, option prices will contain all available information concerning the expected future volatility of underlying prices—including any information used by ARCH models—and thus volatility forecasts implied by option prices should encompass volatility forecasts from ARCH models. However, most studies find that option-implied volatility cannot encompass ARCH in one-day-ahead volatility forecasting and thus conclude that either the option market is not efficient or the option-pricing models employed are misspecified or are at least problematic for short-term forecasting (e.g., see Day and Lewis 1992).[1] We find that option-implied volatility can encompass a simple ARCH model at one-day and one-month horizons, but when the effects of lagged trading volume and option-implied volatility are incorporated in an augmented ARCH model, an implied volatility forecast is encompassed by this broader time-series augmented information set. Treating relative trading volume as a proxy for changes in the set of information available to investors,[2] our findings reveal an important switching role for trading volume between a volatility forecast that reflects relatively stale information (the historical ARCH estimate) and the option-implied forward-looking estimate. With trading volume low relative to the recent past, ARCH is weighted

[1]Note that Christensen and Prabhala (1998) and Fleming (1998) find that option-implied volatility can outperform ARCH at longer horizons (e.g., one-month-ahead forecasts in Christensen and Prabhala 1998) once certain biases are accounted for (as in Fleming 1998). Day and Lewis (1993) argue that, applied to the crude oil futures market, options-implied volatility appears to subsume the information contained in other volatility forecasts at the near-term horizon, though their near term is 32.5 trading days on average, not 1 day.

[2]We thank the referee for pointing out this interpretation of relative trading volume.

more heavily in an augmented forecasting model than options for forecasting future stock market volatility; conversely, with volume high relative to the previous recent past, option-implied volatility is weighted more heavily than ARCH for forecasting future volatility.

## II. Basic Models and Data

The data we employ in this study range from 1988 to 2003, analyzed at the daily and monthly frequency. The stock returns come from the S&P 100 index and option-implied volatilities come from the Chicago Board of Exchange's (CBOE) VIX index.[3] Model selection is performed on in-sample data from 1988:1 to 1995:9, and data from 1995:10 to 2003:8 are our holdout sample for out-of-sample forecast evaluation.[4]

Define stock returns, $R_t$, as the arithmetic return based on the daily closing value of the S&P 100 index, multiplied by 100. Various specification tests on the daily S&P 100 data after 1987 (which is the period we study because index option markets were thin before 1988) reveal that expected returns are appropriately modeled with a constant. As a baseline in our own investigations, we therefore employ the constant expected returns specification.[5] The simplest model we consider, the naive model, has a constant mean and constant variance, forecasting variance as the average variance from the in-sample period.

*ARCH*

In the ARCH family of models, volatility forecasts traditionally use only the history of $\varepsilon$. Lagged $\varepsilon^2$ are included to capture volatility clustering; that is, future volatility is related to lagged squared return innovations. Levels of lagged $\varepsilon$ are also sometimes employed to capture the perception that volatility may be related in an asymmetric way to lagged return innovations, with sharp drops in stock prices causing more future volatility than upturns cause. One specification that nests the popular generalized ARCH (GARCH) model of Bollerslev (1986)

[3]The CBOE recently redefined its VIX index. We use the original series based on S&P 100 index prices. See Whaley (2000) for a discussion of the VIX index construction.

[4]In the holdout sample, we measure realized volatility for volatility forecast evaluation as the squared residual from a model of the mean and the variance with both constant, where the constants are measured with data up to but not including the forecast period. There are virtually no differences to using the squared residual from any other model we discuss later.

[5]For robustness, we investigated various alternative specifications for the expected return, including specifications in which expected returns were modeled as a simple AR(1) process, or as a more complicated seasonal process with dummy variables for January and Monday, plus AR terms as necessary to completely whiten the in-sample data. Our volatility results did not change appreciably. We therefore report the simple constant specification in the following analysis.

and allows asymmetric responses to lagged return shocks is the asymmetric sign-GARCH model of Glosten, Jagannathan, and Runkle (1993), shown in equations (1) to (3):

$$R_t = \mu + \varepsilon_t; \varepsilon_t \sim \left(0, h_t^2\right), \tag{1}$$

$$h_t^2 = \alpha + \beta h_{t-1}^2 + \gamma \varepsilon_{t-1}^2 + \delta D_{t-1} \varepsilon_{t-1}^2, \tag{2}$$

$$D_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

A Glosten, Jagannathan, and Runkle (1993) model with $p$ lags of $h_t$, $q$ lags of $\varepsilon_t^2$, and $r$ lags of $D_t\varepsilon_t^2$ is labeled GJR(*p, q, r*). Such a model excluding the asymmetric volatility term $D_t\varepsilon_t^2$ is labeled GARCH(*p, q*). For simplicity, models in the ARCH family are referred to simply as ARCH when there is no ambiguity.

In this article, the ARCH model we use is the lowest order model that removed evidence of residual autocorrelation, ARCH, and sign-ARCH effects. With our data and period, the very simple GARCH(1,1) was sufficient and is ranked best by the Schwarz information criteria.[6] The GARCH(1,1) model uses one lag each of $h^2$ and $\varepsilon_t^2$ in equation (2). The models we rank are all of the form in equations (1) to (3), with as many as two lags of $h^2$, $\varepsilon^2$, and the asymmetric volatility term. Details on the full set of various alternative models estimated can be found at www.markkamstra.com.

We report parameter estimates for the GARCH(1,1) model, estimated with maximum likelihood assuming normality, in section IV, along with comparisons with other models. We present standard errors based on Bollerslev and Wooldridge (1992), which are robust to nonnormality. Results from various in-sample and out-of-sample diagnostic tests and performance evaluations are also presented in sections IV and V.

*Option-Implied Volatility*

We use the CBOE VIX index to measure option-implied volatility. There is a maturity mismatch when using the VIX for volatility forecasting over horizons less than one month, given the VIX is designed to forecast volatility over a

[6]The sign-ARCH test probes for asymmetric volatility increases from negative shocks to the returns process. See Engle and Ng (1993) for details on the sign-ARCH tests. The Schwarz information criterion is widely used in the literature (e.g., Nelson 1991 uses Schwarz to select exponential GARCH (EGARCH) models). As noted by Nelson (1991), the asymptotic properties of this criterion are unknown in the context of selecting ARCH models; hence, we might also rely on the principle of parsimony among models that do not fail common specification tests to pick the favored model. This would lead us again to the GARCH(1,1) model.

22-business-day period (one calendar month, approximately). Although the VIX volatility forecast can be interpreted as the average volatility over the coming month, and hence a measure of the volatility over the coming day, at least two outcomes are likely. One, the VIX can be expected to be less efficient at forecasting volatility one day rather than one month ahead. Two, forecast errors from the VIX will be correlated over the month-long period that the VIX forecasts overlap. As a result, we consider the performance of our volatility forecasting techniques over both daily and monthly periods. Also, when comparing the VIX with other forecasts of volatility on daily data, we employ econometric techniques designed to be robust to a 22-day moving average forecast error process.

*Volume*

The final variable we consider is trading volume at the New York Stock Exchange (NYSE). Because our objective is to forecast volatility, we are interested in lagged volume, that is, $Volume_{t-1}$, or some function of $Volume_{t-1}$. Because one purpose of our investigation is to determine whether ARCH and options behave differently on high- versus low-volume days, we first consider the high/low volume indicator variable $V_{t-1}$, where:

$$V_{t-1} = \begin{cases} 1 & \text{if } Volume_{t-1} \geq \dfrac{1}{(n-1)} \displaystyle\sum_{i=2}^{n} Volume_{t-i} \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

We set $n = 5$ so that $V_{t-1}$ equals 1 if lagged volume is above its one-week lagged moving average (in the case of daily data), and 0 otherwise. We find no qualitative difference when considering other lag lengths, including a one-month lagged moving average instead of one week for daily data. We form this volume variable relative to the past five months of data for the monthly forecasting exercises.

## III. Combining Forecasts

*Combining Model and Results*

Perhaps the most obvious way to isolate and compare ARCH, option, and volume effects is to estimate a simple linear combination of the ARCH forecasts and option-implied forecasts, along with our high/low volume-switching variable. Combining has a long tradition in the forecasting literature (e.g., see Clemens 1989). We first use equations (1) to (3) and returns information from period $t-1$ to calculate an ARCH volatility forecast for period $t$, and we denote this conditional volatility forecast $\hat{h}_t^2$. We next use the VIX implied volatility estimate (formed in period $t-1$), denoted $\hat{S}_t^2$. We then calculate our volume variable, $V_{t-1}$, from

**TABLE 1. Parameter Estimates from the Combining Regression.**

| $\hat{\sigma}_t^2 =$ | 0.009 | $-$ | $0.034V_{t-1}$ | $+$ | $0.364\hat{h}_t^2$ | $-$ | $0.74V_{t-1}\hat{h}_t^2$ | $+$ | $0.309\hat{S}_t^2$ | $+$ | $0.406V_{t-1}\hat{S}_t^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | {.100} | | {0.228} | | {1.54} | | {−2.21**} | | {2.38**} | | {1.89*} |
| | (0.016) | | (0.147) | | (0.237) | | (0.335) | | (0.13) | | (0.215) |

Note: The variables are defined as follows:
$\hat{\sigma}_t^2 =$ stock market return volatility on period $t$;
$V_{t-1} =$ indicator variable indicating that trading volume is higher than average over the past week;
$\hat{h}_t^2 =$ forecasted volatility from an autrogressive conditional heteroskedasticity (ARCH) model, conditional on $t-1$ information; and
$\hat{S}_t^2 =$ forecasted volatility from option prices, conditional on $t-1$ information.

Log likelihood $= -2269.69$, Bayesian information criterion $= 4600.026$. The jointly estimated mean parameter from equation (1) is $\mu = 0.043$, with standard error of 0.016. There is little or no evidence of autocorrelation, ARCH or sign-ARCH. The $R^2$ is 0, as the mean is modeled with only a constant term. A more parsimonious version of this model is presented in section IV, and more detailed diagnostics are presented there. Bollerslev-Wooldridge (1992) robust two-sided $t$-tests are in brackets, and robust standard errors are in parentheses.

***Significant at the 1% level, two-tailed test.
**Significant at the 5% level, two-tailed test.
*Significant at the 10% level, two-tailed test.

equation (4). Finally, we combine these three variables in a joint mean-variance maximum likelihood regression to obtain our combined volatility forecast, $\hat{\sigma}_t^2$.

$$\hat{\sigma}_\tau^2 = \alpha_0 + \alpha_1 V_{t-1} + \phi_{ARCH,0}\hat{h}_{t-1}^2 + \phi_{ARCH,1} V_{t-1}\hat{\sigma}_{t-1}^2 + \phi_{Option,0}\hat{S}_{t-1}^2 + \phi_{Option,1} V_{t-1}\hat{S}_{t-1}^2, \quad (5)$$

where $V_{t-1}$ is defined in equation (4). We report parameter estimates from the combining regression equation (5) in Table 1. The results in Table 1 are daily for January 1988 to September 1995, for a total of 1,959 observations.

From the theoretical studies on trading volume cited in the introduction and discussed more fully later (e.g., Admati and Pfleiderer 1988), we expect market prices to be more informative during high-volume periods and thus would expect option-implied volatility, which is based on market prices, to forecast volatility more accurately, whereas ARCH models based on a long history of lagged prices would not necessarily prove helpful in forecasting. The parameter estimates we find are consistent with this expectation. When volume is light relative to the past week, the weights assigned to the ARCH forecast and the option-implied forecast are roughly equal, with coefficient estimates of 0.309 on the option-implied term and 0.364 on the ARCH term (although the ARCH coefficient estimate is not significant at the 10% level). However, when volume is heavy relative to the past week, the weight on the option-implied forecast more than doubles to 0.715 (the sum of the coefficient estimates on $\hat{S}_t^2$ and $V_{t-1}\hat{S}_t^2$) and the weight on the ARCH forecast now becomes negative, $-0.376$ (the sum of the coefficient estimates on $\hat{h}_t^2$ and $V_{t-1}\hat{h}_t^2$).

We conduct several tests to check the robustness of this result. First, we investigate different definitions of volume (e.g., equation (4) based on a one-month lag instead of a one-week lag), different data series of option-implied volatility and returns (based on the S&P 500), and different definitions of ARCH and the mean equation (e.g., different models, different lag lengths, etc.). The resulting regression coefficient estimates change somewhat from specification to specification, but the basic finding remains: option-implied volatility dominates ARCH in high-volume states, and ARCH matches or dominates option-implied volatility in low-volume states.

Second, we separate our data into various subsamples and repeat the regression. Again, coefficient estimates change somewhat from subperiod to subperiod, but the key result is qualitatively robust.

Third, instead of using the variance of the residual from equation (1) as the dependent variable in the combining regression, we define the dependent variable as option-implied volatility from day $t$. (Thus, we use ARCH, volume, and option-implied volatility on day $t-1$ to forecast option-implied volatility on day $t$.) Here again we find our familiar result: option-implied volatility is a better forecaster of future volatility relative to ARCH when volume is high, no matter how the term "volatility" is defined. Regardless of the specification for volatility, this core result is robust. We also considered conducting various option-based tests on our volatility forecasts, including tests for pricing/hedging effectiveness, but we did not do so because pricing/hedging effectiveness testing cannot be legitimately undertaken on the volume-ARCH-option combinations we study given the menu of option pricing models currently in existence.

*Discussion*

As stated in the introduction, we are not attempting to test market efficiency in our study. It is, however, interesting to note that on days that have high volume relative to the past week, days we expect to exhibit changes in the investor information set, option-implied volatility dominates ARCH. Thus, using the yardstick suggested by other authors, the market may indeed be efficient when enough information is flowing into the market (assuming volume is a good proxy for information flow, as is frequently assumed; e.g., Admati and Pfleiderer 1988). The failure of option-implied volatility to dominate ARCH on low-volume days might suggest that if the market is indeed inefficient, it may only be so when there is comparatively little information flowing. Alternatively, our results could be interpreted to reveal that the Black and Scholes (1973) model is misspecified in some way that is most clearly seen on low-volume days and that the market is always efficient.

There appear to be at least two possible (not necessarily exclusive) explanations for our finding that option-implied volatility provides a better volatility forecast relative to ARCH following high-volume days: (1) the informativeness of the ARCH volatility forecast declines in high-volume states, and/or (2) the informativeness of option-implied volatility increases in high-volume states.

We investigate whether there is any change in average volatility following high- versus low-volume days as this could lead ARCH to underforecast future volatility following high-volume days and thus help explain why ARCH does worse relative to options following high-volume days. To examine this possibility, we compare average squared errors from equation (1) and average volatility forecasts from our ARCH and option-implied models on day $t$ when volume was high versus low on day $t-1$. We find, based on our in-sample data 1988 to September 1995, that when volume was high on day $t-1$, the average day $t$ squared error is 0.724, and when volume was low on day $t-1$, it is 0.721. Moreover, when volume was high on day $t-1$, the average day $t$ deviation between the ARCH and option-implied forecasts shrinks relative to the deviation following relatively low-volume periods. In other words, squared pricing errors are almost identical following high- and low-volume days, and the ARCH model matches option-implied volatility better on days following high volume. The close match of average squared pricing errors on high- and low-volume days is revealed in the insignificant intercepts in the combining regression in Table 1. This suggests the ARCH versus options effect is not coming from average volatility levels, and thus any explanation of our results is more likely to rest on intertemporal correlations between forecasted and realized volatility.

To investigate correlation effects, we compute the simple correlation between realized volatility at time $t$, as measured by the time $t$ squared return innovation, and the ARCH (option-implied) volatility forecast at time $t-1$ based on our in-sample data, January 1988 to September 1995. This correlation equals 24.8% (26.0%) when volume is low on day $t-1$ and is 15.8% (23.1%) when volume is high on day $t-1$. On the full sample, January 1988 to August 2003, we observe a similar pattern, with the correlation between realized volatility at time $t$ and the ARCH (option-implied) volatility forecast at time $t-1$ equaling 33.4% (34.5%) when volume is low on day $t-1$ and 26.8% (35.4%) when volume on day $t-1$ is high. In other words, ARCH volatility works best following a low-volume period, and option-implied volatility is roughly as good following high- or low-volume periods (relative to recent volume levels). It therefore seems likely that the ARCH versus options effect is at least partially driven by ARCH doing worse following high-volume days versus low-volume days and option-implied volatility doing as well on either high- or low-volume days. The conditional analysis and formal tests that follow suggest further that the benefit derived from the time-series information embedded in ARCH and volume come mainly from improving forecasts following low-volume states, with the weight on time-series information changing to zero or negative values following high-volume periods.

## IV. Augmented ARCH

Forecast combining was appropriate in the previous section because our purpose was to reveal the basic ARCH-volume-option relation. Forecast combining is also a useful tool when the econometrician possesses the forecasts produced by various models but not the information sets used to produce the forecasts. However, we do possess the information set on which at least the ARCH forecasts are based; thus, to produce optimal volatility forecasts, we should ideally add option and volume information to the ARCH model directly and estimate an augmented ARCH mega-model. We therefore investigate augmented ARCH models in this section.

*Model*

The augmented ARCH model we employ is given in (6), in which $R_t$ is the daily arithmetic stock return (multiplied by 100) and $S_t^2$ is option-implied return variance.

$$R_t = \mu + \varepsilon_t; \varepsilon_t \sim \left(0, \sigma_t^2\right) \tag{6}$$

$$\begin{aligned} \sigma_t^2 = {} & \alpha_0 + \alpha_1 V_{t-1} + \beta_0 \sigma_{t-1}^2 + \beta_1 V_{t-1} \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \\ & + \phi_{Option,0} S_{t-1}^2 + \phi_{Option,1} V_{t-1} S_{t-1}^2, \end{aligned} \tag{7}$$

where $V_{t-1}$ is defined in equation (4), and equations (6) and (7) are estimated jointly under maximum likelihood with $n = 5$, as are several restricted and extended versions of (6) and (7). Results are reported next.

To understand better the effects of adding lagged volume and implied volatilities to ARCH, we investigate many possible combinations and permutations within our model, including: each variable alone, each possible combination, variables interacting with each other, and so forth. We also expand our model to investigate a variety of different functional forms for the conditional volatility, including variables, and groups of variables, added and interacted nonlinearly. Table 2 presents daily data in-sample estimation results for a small collection of models, which reveal the most interesting information concerning the effects of volume and implied volatility, for daily data from January 1988 to September 1995. Summary statistics on daily and monthly data for these models follow in Table 3, Panels A and B. In all cases our core result—that options provide better forecasts relative to ARCH on high-volume days than on low-volume days—remains qualitatively robust.

*Parameter Estimates*

In Panel A of Table 2 we report parameter estimates (with Bollerslev-Wooldridge robust standard errors in parentheses) for the most interesting specifications contained within equations (5) through (7). In Panel B we report common diagnostics for each model. These diagnostics include: the model log likelihood; the Bayesian information criterion (BIC); the $p$-value from a test for residual autocorrelation (Wald AR, a $\chi^2$ Wald test using 5 lags of the residual from the mean equation); the $p$-value from a traditional Ljung-Box (1978) test for symmetric ARCH at 24 lags; and the $p$-values from an Engle-Ng (1993) sign bias test, negative sign bias test, positive sign bias test, and joint sign bias test—all at 5 lags—for the presence of asymmetric ARCH effects.

We begin our analysis of Table 2 by considering the results from the basic GARCH(1,1) specification, as reported in column 3 (labeled ARCH). Note from Panel A, column 3 that all the parameter estimates from the ARCH model are of the expected sign and magnitude and, from Panel B, that the model passes all standard specification tests at conventional significance levels (e.g., there are no $p$-values below .050 in Panel B, column 2). Note in particular from Panel A that

**TABLE 2. Estimation Results and Diagnostics for Volatility Models Based on Equations (5) Through (7) and Daily Data.**

| Parameter | Naive | Options Only | ARCH Model | Combining Model | Full MLE |
|---|---|---|---|---|---|
| Panel A. Parameter Estimates (Robust Standard Error) | | | | | |
| $\mu$ | 0.047** | 0.042** | 0.047*** | 0.043*** | 0.045*** |
| | (0.019) | (0.016) | (0.016) | (0.016) | (0.016) |
| $\alpha_0$ | 0.723*** | 0.010 | 0.002 | 0.019 | 0.032*** |
| | (0.049) | (0.050) | (0.001) | (0.053) | (0.010) |
| $\beta_0$ | | | 0.980*** | | 0.842*** |
| | | | (0.008) | | (0.114) |
| $\beta_1$ | | | | | −1.31*** |
| | | | | | (0.336) |
| $\gamma$ | | | 0.016** | | −0.016 |
| | | | (0.007) | | (0.014) |
| $\phi_{Option,0}$ | | 0.519*** | | 0.307** | 0.049 |
| | | (0.059) | | (0.130) | (0.055) |
| $\phi_{Option,1}$ | | | | 0.411* | 0.709*** |
| | | | | (0.212) | (0.184) |
| $\phi_{ARCH,0}$ | | | | 0.350 | |
| | | | | (0.222) | |
| $\phi_{ARCH,1}$ | | | | −0.706** | |
| | | | | (0.330) | |
| Panel B. Diagnostics | | | | | |
| Log likelihood | −2461.41 | −2274.27 | −2311.62 | −2269.75 | −2269.71 |
| BIC | 4945.565 | 4578.857 | 4661.135 | 4592.553 | 4600.070 |
| Wald AR test | 0.119 | 0.133 | 0.124 | 0.095 | 0.071 |
| LM ARCH test | 0.000 | 0.965 | 0.980 | 0.968 | 0.965 |
| Sign test | 0.529 | 0.609 | 0.521 | 0.639 | 0.639 |
| Neg. sign test | 0.000 | 0.851 | 0.446 | 0.792 | 0.801 |

(Continued)

**TABLE 2. Continued.**

| Parameter | Naive | Options Only | ARCH Model | Combining Model | Full MLE |
|---|---|---|---|---|---|
| Pos. sign test | 0.000 | 0.063 | 0.059 | 0.042 | 0.100 |
| Joint sign test | 0.000 | 0.612 | 0.224 | 0.485 | 0.555 |

Note: Panel A presents coefficient estimates for the variance forecasting models.

$$\hat{\sigma}_t^2 = \alpha_0 + \alpha_1 V_{t-1} + \phi_{ARCH,0}\hat{h}_{t-1}^2 + \phi_{ARCH,1} V_{t-1}\hat{\sigma}_{t-1}^2 + \phi_{Option,0}\hat{S}_{t-1}^2 + \phi_{Option,1} V_{t-1}\hat{S}_{t-1}^2$$

or

$$\hat{\sigma}_t^2 = \alpha_0 + \alpha_1 V_{t-1} + \beta_0\hat{\sigma}_{t-1}^2 + \beta_1 V_{t-1}\hat{\sigma}_{t-1}^2 + \gamma\hat{\varepsilon}_{t-1}^2 + \phi_{Option,0}\hat{S}_{t-1}^2 + \phi_{Option,1} V_{t-1}\hat{S}_{t-1}^2,$$

where

$\hat{\sigma}_t^2$ = stock market return volatility on period $t$;
$V_{t-1}$ = indicator variable indicating that trading volume is higher than average over the past week;
$\hat{h}_t^2$ = forecasted volatility from an ARCH model, conditional on $t-1$ information;
$\hat{S}_t^2$ = forecasted volatility from option prices, conditional on $t-1$ information;
$R_t = \mu + \varepsilon_t; \varepsilon_t \sim (0, \sigma_t^2)$, where $R_t$ is the daily arithmetic stock return (multiplied by 100); and
$\hat{\varepsilon}_t = R_t - \hat{\mu}$, where $\hat{\mu}$ is the estimate of the mean return from the naive model.

Panel B reports common diagnostics for each model. These diagnostics include: the model log likelihood, the Bayesian information criterion (BIC), the $p$-value from a test for residual autocorrelation (Wald AR = a $\chi^2$ Wald test on 5 lags of the residual in the mean equation), the $p$-value from a traditional Ljung-Box (1978) test for symmetric ARCH at 24 lags, and the $p$-values from an Engle-Ng (1993) sign bias test, negative sign bias test, positive sign bias test, and joint sign bias test—all at 5 lags—for the presence of asymmetric ARCH effects.

***Significant at the 1% level, two-tailed test.
**Significant at the 5% level, two-tailed test.
*Significant at the 10% level, two-tailed test.

the parameter on lagged conditional volatility, $\beta_0$, is close to unity, which reveals the highly persistent nature of stock return volatility.

In column 1 (naive) of Table 2 we report results from a constant mean and constant variance model, forecasting next-period variance as the average variance from the in-sample period. This model displays gross evidence of misspecification with very strong residual ARCH and sign-ARCH effects, though no evidence of autocorrelation.

In column 2 (options only) of Table 2 we report results from option-implied volatility alone, that is, results from equations (6) and (7) estimated with all parameters set to zero except for the intercepts and $\phi_{Option,0}$. Note in Panel B, column 2 that the log likelihood from option-implied volatility alone exceeds the log likelihood from ARCH in column 3. Also note in Panel B that option-implied volatility passes all of the ARCH tests at the 5% significance level.

In column 4 (combining model) of Table 2 we report results from the combining regression model of equation (5), constraining the volatility intercept dummy variable $V_{t-1}$ to have a coefficient estimate of zero. This model also removes most evidence of sign-ARCH effects and appears similar to the combining model that includes the intercept volume dummy variable. The log likelihood is improved relative to either the ARCH or option-implied model alone, although the BIC criterion favors the simple option-implied volatility model over the more highly parameterized combining model.

In column 5 (MLE (maximum likelihood estimate)) of Table 2 we report results from adding option-implied volatility and our high/low volume indicator variable to the standard ARCH model, a representative but parsimonious model of the set of models that could be constructed from interacting the volume variable with the various ARCH variables and the option-implied volatility variable. Unreported results reveal that, either alone or when added to a standard ARCH model, lagged volume has no power to predict volatility. From this, one might be tempted to conclude, as previous researchers conclude, that lagged volume has no power to forecast future volatility once the effects of lagged return innovations have been accounted for. However, such a conclusion would be premature. It would be more accurate to argue that although volume cannot by itself forecast volatility, it does play an important regime-switching role, interacting with other variables in the model, as we have already seen. Here, we observe that on high-volume days, the weight on option-implied volatility increases and the weight on the lagged conditional variance decreases (i.e., $\beta_1 < 0$ and $\phi_{Option,1} > 0$). In other words, the full MLE model from Table 2 confirms our findings from the simple combining exercise we reported in Table 1. The results from this model are, however, even more dramatic than those from the simple combining exercise. The weight on the option-implied variable is effectively 0 in the low volume state, equal to 0.049 and statistically insignificant, and the weight on the lagged conditional variance flips to nearly $-0.5$ in the high volume state (relative to the last weeks' average volume). Compared to the combining model reported in Table 2, however, the log likelihood and BIC give little reason to prefer the full MLE model over the combining model on daily data. As presented next, out-of-sample forecast performance confirms this for daily data but shows advantage to the full MLE model for monthly forecast horizons.

Results for the daily data model estimations for the full sample, 1988 to 2003, as well as for monthly data, 1988 to 1995 and the full sample, can be found at www.markkamstra.com. These results are qualitatively identical to the results discussed previously.

*Summary Statistics*

In Table 3, Panels A and B, we present summary statistics on all of the models in Table 2 for daily and monthly data. In the first row in Table 3 (raw data), we report statistics for the S&P 100 returns (recall that returns are arithmetic and multiplied by 100). The row labeled ARCH is for the basic GARCH(1,1) model in equations (1) to (3). Option signifies variance defined as lagged option-implied volatility, that is, equations (6) and (7) with all parameters zero except $\mu$, $\alpha_0$, and $\phi_{Option,0}$, with the implied volatility suitably rescaled for the forecast horizon. The combining model is a modified version of the model presented in section III, in which the volume dummy variable $V_{t-1}$ is omitted, as in column 4 of

**TABLE 3. Summary Statistics.**

Panel A. Daily Data: January 1, 1988, to September 29, 1995

| | | Fitted Variance | | | | Standardized Returns | | |
|---|---|---|---|---|---|---|---|---|
| Method | Mean | Std | Skew | Kurtosis-3 | RMSE | Std | Skew | Kurtosis-3 |
| Raw data | 0.723 | 2.161 | 13.660 | 275.757 | 2.991 | 1.000 | −0.499 | 6.963 |
| ARCH | 0.723 | 0.456 | 1.855 | 4.130 | 2.725 | 1.001 | −0.573 | 5.397 |
| Option | 0.715 | 0.510 | 2.396 | 8.092 | 2.503 | 1.000 | −0.495 | 4.278 |
| Combining | 0.712 | 0.512 | 2.615 | 11.006 | 2.464 | 1.000 | −0.490 | 4.084 |
| Full MLE | 0.707 | 0.507 | 2.774 | 11.709 | 2.470 | 1.000 | −0.470 | 4.113 |

Panel B. Monthly Data: January 1988 to September 1995

| | | Fitted Variance | | | | Standardized Returns | | |
|---|---|---|---|---|---|---|---|---|
| Method | Mean | Std | Skew | Kurtosis-3 | RMSE | Std | Skew | Kurtosis-3 |
| Raw data | 12.353 | 20.892 | 2.637 | 6.764 | 1.691 | 1.000 | −0.051 | 0.944 |
| ARCH | 12.276 | 5.479 | 0.687 | −0.340 | 1.583 | 1.007 | −0.293 | 0.566 |
| Option | 13.468 | 11.181 | 3.012 | 12.220 | 1.514 | 1.005 | −0.245 | 0.364 |
| Combining | 13.117 | 10.041 | 2.459 | 7.507 | 1.448 | 1.005 | −0.157 | 0.151 |
| Full MLE | 12.931 | 10.810 | 2.388 | 7.165 | 1.260 | 1.005 | 0.027 | −0.382 |

Panel C. Daily Data: October 2, 1995 to August 8, 2003

| | | Variance Forecast | | | | Standardized Returns | | |
|---|---|---|---|---|---|---|---|---|
| Method | Mean | Std | Skew | Kurtosis-3 | RMSE | Std | Skew | Kurtosis-3 |
| Raw data | 1.737 | 3.609 | 5.986 | 53.673 | 3.916 | 1.000 | 0.003 | 2.324 |
| Naive | 0.917 | 0.177 | 0.315 | −1.184 | 4.006 | 1.359 | −0.069 | 2.505 |
| ARCH | 1.638 | 1.087 | 1.609 | 3.250 | 2.177 | 1.061 | −0.340 | 1.710 |
| Option | 1.372 | 0.814 | 1.638 | 3.214 | 2.098 | 1.099 | −0.194 | 0.977 |
| Combining | 1.397 | 0.856 | 1.664 | 3.283 | 2.118 | 1.097 | −0.189 | 1.063 |
| Full MLE | 1.363 | 0.815 | 1.655 | 3.318 | 2.148 | 1.107 | −0.195 | 1.029 |

Panel D. Monthly Data: October 1995 to July 2003

| | | Variance Forecast | | | | Standardized Returns | | |
|---|---|---|---|---|---|---|---|---|
| Method | Mean | Std | Skew | Kurtosis-3 | RMSE | Std | Skew | Kurtosis-3 |
| Raw data | 25.947 | 32.242 | 2.211 | 5.093 | 2.150 | 1.000 | −0.049 | −0.453 |
| Naive | 15.063 | 2.873 | 0.275 | −1.471 | 2.268 | 1.315 | −0.010 | −0.412 |
| ARCH | 20.892 | 12.747 | 0.952 | 1.046 | 2.000 | 1.217 | 0.013 | −0.206 |
| Option | 23.262 | 13.901 | 1.650 | 3.303 | 1.466 | 1.107 | −0.247 | −0.619 |
| Combining | 22.541 | 14.862 | 1.752 | 4.513 | 1.993 | 1.185 | −0.411 | −0.100 |
| Full MLE | 23.642 | 17.174 | 1.903 | 5.162 | 1.525 | 1.134 | −0.172 | −0.704 |

Table 2. The last row of each panel reports results from the full MLE, which is the maximum likelihood combination of ARCH, volume, and options obtained by estimating equations (6) and (7), including interaction terms between volume and the option-implied volatility, and volume and the lagged conditional variance, as in

column 5 of Table 2. For in-sample returns, the naive model yields a constant variance forecast and standardized returns equal to the raw data; therefore, results based on that model are not reported.

We consider now the properties of the fitted variance and standardized in-sample returns based on the summary statistics presented in Table 3, Panels A and B. The variance (squared error) forecasts for daily and monthly data all have a mean close to the average squared error—the raw data (the raw option-implied volatility is biased but our estimate corrects this bias by estimating an intercept and slope term, $\alpha_0$ and $\phi_{Option,0}$). All models produce forecasts that are less volatile than the actual squared error, as well as generally less skewed and kurtotic. The root mean squared error (RMSE) column favors the more highly parameterized models. Of particular interest in Table 3, Panels A and B, are results from the columns on standardized returns, that is, $\hat{\varepsilon}_t/\hat{\sigma}_t$. The combining and the full MLE models deliver the lowest kurtosis, and basic ARCH removes the least kurtosis.

## V. Forecasts

A model's ability to forecast future volatility is the most important feature to consider in the present context. To measure realized volatility, we employ the squared residual from the naive model of the mean return. There are virtually no differences to using the residual from any of the other models. We find that the interaction of volume with option-implied and ARCH forecasts produces volatility forecasts that encompass both the option-implied and ARCH forecasts themselves, with the cleanest results being for the monthly data, the forecast horizon for which the option-implied forecasts are designed.

To explore out-of-sample forecasting, we first use data from January 4, 1988, through September 29, 1995, to estimate model parameters. We then use these parameters with the data from January 1988 through September 1995 to produce for October 2, 1995 (the next business day that markets were open) a one-step-ahead out-of-sample forecast of both the level and volatility of the expected return innovation (i.e., both $\hat{\varepsilon}_t$ and $\hat{h}_t$). Next, we update our information set by one period, using data from January 4, 1988, through to October 2, 1995, to produce for October 3, 1995, a one-step-ahead out-of-sample forecast of both the level and volatility of the expected return innovation. This process continues until we obtain one-step-ahead out-of-sample forecasts for October 2, 1995, through August 8, 2003. Tests based on these one-step-ahead out-of-sample forecasts are reported next.

Table 3, Panels C and D present out-of-sample statistics analogous to those presented in Table 3, Panels A and B, but in addition we now include forecasts from the naive model. Most of the models produce out-of-sample normalized returns that are less kurtotic than the raw data for the daily data forecast horizon, except for the naive model. For the monthly forecast horizon there is no excess kurtosis, even for the raw data. All of the models produce one-step-ahead forecasts that are biased downward, so that the normalized returns are too volatile. For both daily and monthly horizons, the option-implied forecasts yield the smallest mean squared (forecast) error. We would expect to observe much lower out-of-sample RMSE for the more complex forecasting techniques relative to that of the naive model, and this is true. The RMSE of the naive model is nearly twice that of all the models on daily data, and as much as 50% larger than the best model on monthly data.

To more carefully evaluate the out-of-sample forecasting performance of each model relative to the other models, we conduct forecast-encompassing tests similar to Chong and Hendry (1986), Fair and Shiller (1990), and Day and Lewis (1992, 1993). To examine forecast encompassing, we estimate a single bivariate regression with both forecasts as regressors and test for the significance of the parameter estimates in equation (8):

$$\hat{\varepsilon}_t^2 = \alpha + \beta_j \hat{h}_{t,j} + \beta_i \hat{h}_{t,i} + \nu_t, \tag{8}$$

in which ($\hat{\varepsilon}_t^2$) is the naive model forecast error,[7] $\hat{h}_{t,j}$ is the model $j$ forecast, $\hat{h}_{t,i}$ is the model $i$ forecast, and $\nu$ is a random error. Multicollinearity can lead to both $\beta$ coefficient estimates being insignificant when equation (8) is estimated, whereas sufficiently nonoverlapping information sets can lead to both estimated $\beta$ coefficients being significant.

The results of estimating equation (8) parameters and standard errors are reported in Table 4, Panels A and B. The estimation of equation (8) is based on Hansen's (1982) generalized method of moments (GMM) and Newey and West's (1987) heteroskedasticity- and autocorrelation-consistent (HAC) covariances, employing 1 lag of regressors as instruments and 22 lags to construct the HAC standard errors for daily data (to correct for the overlapping nature of the option-implied forecast errors). The $R^2$ statistics are based on simple ordinary least squares.

The first block of results in the table is single regressor models for which the out-of-sample squared residuals are regressed one at a time on each model we consider. The remaining blocks of results detail pairwise regressions described by equation (8).

The daily and monthly out-of-sample regressions contained in the first block of Table 4, Panels A and B (the univariate results), provide strong evidence that the naive model is inadequate compared with all the other models, with a large jump in out-of-sample $R^2$ moving from the single regressor naive forecast regression to any of the other forecast regressions (2.2% $R^2$ in Table 4, Panel A for the naive model, first row of results, to 12.1% $R^2$ for the combining model, with even more dramatic improvements for monthly forecasting in Table 4, Panel B). There is a similar, though less extreme, advantage of the remaining models over the simple ARCH model in terms of $R^2$ or total out-of-sample explanatory power. All

[7]Results are not sensitive to this choice.

**TABLE 4. Daily and Monthly Out-of-Sample Regression Results.**

Panel A. Daily Data Out-of-Sample Regression Results

Parameter Estimates (Robust Standard Error)

| Regressors | Naive | ARCH | Options | Combining Model | Full MLE | $R^2$ |
|---|---|---|---|---|---|---|
| Naive | 3.15*** | | | | | .022 |
| | (.432) | | | | | |
| ARCH | | .870*** | | | | .068 |
| | | (.120) | | | | |
| Options | | | 1.45*** | | | .119 |
| | | | (.214) | | | |
| Combining | | | | 1.47*** | | .121 |
| | | | | (.207) | | |
| Full MLE | | | | | 1.51*** | .116 |
| | | | | | (.219) | |
| Naive | .488 | .828*** | | | | .068 |
| and ARCH | (.485) | (.132) | | | | |
| Naive | −0.44 | | 1.56*** | | | .120 |
| and options | (.625) | | (.238) | | | |
| Naive | −0.86 | | | 1.66*** | | .122 |
| and combining | (.691) | | | (.251) | | |
| Naive | −0.52 | | | | 1.64*** | .117 |
| and full MLE | (.648) | | | | (.258) | |
| ARCH | | −0.14 | 1.65*** | | | .120 |
| and options | | (.215) | (.388) | | | |
| ARCH | | −0.11 | | 1.50*** | | .125 |
| and combining | | (.238) | | (.420) | | |
| ARCH | | −0.10 | | | 1.65*** | .116 |
| and full MLE | | (.202) | | | (.391) | |
| Options | | | .772 | .637 | | .121 |
| and combining | | | (.621) | (.620) | | |
| Options | | | 2.84** | | −1.5 | .120 |
| and full MLE | | | (1.26) | | (1.25) | |
| Combining | | | | 1.45** | .023 | .121 |
| and full MLE | | | | (.730) | (.701) | |
| **Panel B. Monthly Data Out-of-Sample Regression Results** | | | | | | |
| Naive | 1.00 | | | | | .022 |
| | (.952) | | | | | |
| ARCH | | .387 | | | | .053 |
| | | (.287) | | | | |
| Options | | | .539 | | | .071 |
| | | | (.333) | | | |
| Combining | | | | .673** | | .097 |
| | | | | (.338) | | |
| Full MLE | | | | | .661** | .137 |
| | | | | | (.290) | |
| Naive | .051 | .347 | | | | .055 |
| and ARCH | (1.48) | (.425) | | | | |
| Naive | −0.58 | | .714 | | | .071 |
| and options | (1.55) | | (.461) | | | |

(Continued)

**TABLE 4. Continued.**

Panel B. Monthly Data Out-of-Sample Regression Results

Parameter Estimates (Robust Standard Error)

| Regressors | Naive | ARCH | Options | Combining Model | Full MLE | $R^2$ |
|---|---|---|---|---|---|---|
| Naive | −0.67 | | | .781* | | .097 |
| and combining | (1.49) | | | (.438) | | |
| Naive | −0.27 | | | | .722** | .137 |
| and full MLE | (1.23) | | | | (.340) | |
| ARCH | | .074 | .526 | | | .081 |
| and options | | (.399) | (.477) | | | |
| ARCH | | .004 | | .599 | | .101 |
| and combining | | (.451) | | (.502) | | |
| ARCH | | .029 | | | .654* | .145 |
| and full MLE | | (.323) | | | (.356) | |
| Options | | | −0.09 | .531 | | .099 |
| and combining | | | (.478) | (.581) | | |
| Options | | | −0.72 | | 1.20** | .158 |
| and full MLE | | | (.527) | | (.530) | |
| Combining | | | | −0.15 | .747* | .139 |
| and full MLE | | | | (.395) | (.401) | |

Note: Presented here are the parameter estimates from bivariate regressions on the daily and monthly data with both forecasts as regressors, as well tests for the significance of the parameter estimates in the following regression:

$$\hat{\varepsilon}_t^2 = \alpha + \beta_j \hat{h}_{t,j} + \beta_i \hat{h}_{t,i} + \nu_t,$$

where ($\hat{\varepsilon}_t^2$) is the forecast error of the naive model (results are not sensitive to this choice), $\hat{h}_{t,j}$ is the forecast for model $j$, $\hat{h}_{t,i}$ is the forecast for model $i$, and $\nu$ is a random error. Regression coefficient estimates, standard errors, and $p$-values are based on generalized method of moments estimation and heteroskedasticity- and autocorrelation-consistent covariances. The $R^2$ is based on a simple ordinary least squares.

***Significant at the 1% level, two-tailed test.
**Significant at the 5% level, two-tailed test.
*Significant at the 10% level, two-tailed test.

the forecasts are individually significant at the daily forecast horizon,[8] but only the combining and full MLE forecasts are individually significant for at the monthly horizon.

When we consider the first set of bivariate regressions, the second block of Table 4, Panels A and B, we observe that the addition of the naive model forecast to any of the other methods' forecasts does little to increase the explanatory power

[8]There is also evidence that most of the forecasts are biased, with slope coefficient estimates significantly different from 1 and intercepts different from 0 (by inspection, not reported) echoing the results of research such as Day and Lewis (1993). As the relative efficiency of different forecast methods is our focus, this issue is not pursued here.

of the regression at both the daily and monthly forecast horizons. Also, the $\beta$ coefficient estimate on the naive forecast is greatly reduced in magnitude, from more than 3 to less than 1 in absolute magnitude; is negative when included with all but the ARCH forecast; and is statistically insignificant in all cases. In the bivariate regressions of the second block of Table 4, Panels A and B, all but the naive forecast have statistically significant regression coefficient estimates at the daily forecast horizon, but only the combining and full MLE forecasts are significant at the monthly horizon. Together these results suggest that the naive model is easily encompassed by the remaining models, which is consistent with volatility being predictable.

The second set of bivariate regressions, in the third block of Table 4, Panels A and B, permits us to explore the incremental value of the time-series ARCH forecast for out-of-sample returns. For both the daily and monthly horizons, adding ARCH to any of the remaining forecasts, options, combining or full MLE, increases the $R^2$ very little relative to not including ARCH. Most telling, the coefficient estimate on the ARCH forecast is not only statistically insignificant (which it might be simply because of multicollinearity) but also very near zero and greatly reduced in magnitude from the regression with ARCH only.

These results suggest that simple univariate time-series information has little to add relative to the information embedded in options prices, even when we consider a daily forecast horizon with its concurrent mismatch of forecast horizon relative to the option-implied volatility forecast for a monthly horizon. Together these results suggest that the ARCH model is encompassed by the options, combining and full MLE models.

The third set of bivariate regressions, in the fourth block of Table 4 Panel A, reveal that at the daily horizon there is too much multicollinearity between the options forecast of volatility and the combining and full MLE models (models that incorporate the option-implied volatility with times-series information) to discriminate between them. The leveraged coefficient estimates produced when options and full MLE are included together (options having a coefficient estimate of 2.84 and full MLE of $-1.5$) are not particularly meaningful, as virtually no increase in the $R^2$ is associated with this pairing relative to either forecast used individually. Similarly, in the fifth block of Table 4, Panel A (daily data), the pairing of the combining model and full MLE barely changes the $R^2$ relative to either forecast individually. Overall, this suggests there may be no value to the volume interaction variable and ARCH time-series forecast when using daily data, even though for in-sample data the volume interaction terms appear significant. It may also be true that we do not have a long enough sample of daily out-of-sample data to uncover a significant effect from volume and ARCH.

The monthly data horizon is more definitive for determining the incremental value of the new time-series information introduced in this article, the volume data, relative to information embedded in the option-implied volatility forecast.

Consider the third set of bivariate regressions, the fourth block of Table 4, Panel B. Here, we find that adding either the combining forecast or the full MLE forecast to the options forecast drives down the magnitude of the options forecast coefficient estimate and drives out the statistical significance of the coefficient estimate. As well, the $R^2$ of the models using volume information, combining and full MLE, is much more than the remaining models that use only univariate time-series information or option-implied volatility. The full MLE model, dynamically estimating the univariate time-series coefficients, the volume interaction coefficients, and the option-implied volatility coefficient, demonstrates a large advantage for out-of-sample data relative to all of the other models, as it is the only model that has a statistically significant coefficient estimate regardless of what other forecast is added to it, and it is the only model that does not have its coefficient estimate driven down to zero or negative values by any other forecast.

## VI. Summary and Conclusions

Brooks (1998) and Heimstra and Jones (1994), among others, report that lagged trading volume has little or no value to forecast return volatility. In this article we provide a simple model with predictive power for forecasting return volatility, with volume acting as a switching variable between states in which option-implied volatility is more or less informative than ARCH for volatility forecasting, perhaps reflecting important changes in information attendant with increases in trading volume on the NYSE. We find that the accuracy of volatility forecasts can be significantly improved by accounting for the volume effect and by combining information from ARCH models and option prices accordingly, most markedly at a monthly rather than a daily forecast horizon.

Results produced by our investigation reveal that if trading volume was lower than normal during period $t-1$, the best forecast of time $t$ volatility is found by combining the ARCH forecast with the option-implied volatility forecast, with similar weight being given to ARCH and options. Conversely, if trading volume was higher than normal during period $t-1$, the best forecast of time $t$ volatility is obtained by placing more weight on options and less on ARCH. This result is robust to a variety of perturbations of the in-sample period and model specification, and seems to be largely driven by a decline in the quality of the ARCH forecast in absolute terms as well as relative to the option-implied volatility forecast during high-volume periods.

Altogether, this suggests that market prices contain more information relative to historical sources in high-volume periods than in low-volume periods (indeed, our work suggests a new way to test the relative informativeness of market prices in various volume regimes). Our results also suggest either that option markets are more efficient in high-volume periods (that prices encompass historical information only in high-volume periods) or that option pricing models are less misspecified in high-volume periods. This suggests that researchers modeling ARCH may profit from expanding the traditional ARCH information set to include volume, options, and other types of information in addition to the history of lagged return innovations.

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