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# Combining Algorithms Based on Robust Estimation Techniques and Co-integrating Restrictions

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## ABSTRACT

This paper will motivate alternative combining schemes, provide a statistic for comparing alternative combinations out-of-sample and provide an example demonstrating these techniques. The evidence suggests the proposed procedures are likely to do no worse than other approaches and promise to do better under circumstances commonly encountered with economic data: integrated series and contaminated data.

KEY WORDS Combining forecasts Robust estimation Co-integration Encompassing

# INTRODUCTION

Since the Bates-Granger (1969) seminal paper on combining forecasts, new work in the area has come in fits and starts. The Granger-Ramanathan (1984) paper initiated the recent flurry of work, including that of Clemen (1986), Diebold and Pauly (1986), and Holden and Peel (1986). Granger and Ramanathan show that the variance-covariance formula of Bates and Granger implied a regression of the forecasted variable on the forecasts with the sum of the forecast coefficients being restricted to 1 and the intercept term suppressed. It may or may not be appropriate to impose these restrictions in particular cases. Of course, the within-sample mean squared error is reduced when they are not, but the results of Clemen (1986) and Holden and Peel (1986) demonstrate a possible reduction in post-sample mean squared forecast error when the restrictions are imposed.

Granger and Ramanathan also point out that the combined forecast's error may be serially correlated, even when the individual forecasts' errors are not. Improved forecasts may result from a dynamic combination which takes account of this possibility, as in Diebold (1985).

In the spirit of Granger and Ramanathan, this paper views the task of forecast combination as a statistical problem for which standard statistical techniques and approaches are 0277-6693/89/030189-10\$05.00 © 1989 by John Wiley & Sons, Ltd. *Revised March 1988*  appropriate. Specifically, this motivates three observations:

- (1) Forecasted data are often integrated (non-stationary), and this has implications for the form of a linear combination of the forecasts.
- (2) Robust estimation of the combining weights may prove fruitful.
- (3) A ranking rule alternative to minimum mean square error is suggested by considering various combining schemes as statistical models of the variable to be forecasted.

#### FORECASTING INTEGRATED SERIES

When a series  $x_t$  must be differenced d times before it appears to be stationary it is said to be integrated of order d, denoted  $x_t \sim I(d)$ . A vector  $x_t = (x_{1t}, x_{2t}, ..., x_{nt})'$  is I(d) if each of its components is. In general, linear combinations of I(d) variables are also I(d), but it is possible that there exist one or more linear combinations that are integrated of lower order. When this happens,  $x_t$  is said to be *co-integrated*.

As a simple example, suppose  $x_t$  is generated by

$$\begin{aligned} x_{1t} &= x_{1,t-1} + \varepsilon_{1t} \\ x_{2t} &= x_{1t} + \varepsilon_{2t} \end{aligned}$$
 (1)

where  $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$  is a white-noise process. Then both components of  $x_t$  are I(1) but the linear combination

$$x_{1t} - x_{2t} = \varepsilon_{1t} - \varepsilon_{2t}$$

is I(0).

If  $x_t$  is I(1) but there exists a full-rank  $n \times r$  matrix  $\alpha$  such that

$$z_t = (z_{1t}, \dots, z_{rt})' = \alpha' x_t$$
(2)

is I(0), then  $x_t$  is co-integrated with r linearly independent co-integrating vectors  $\alpha_1, \alpha_2, ..., \alpha_r$ . If  $x_t$  is I(1) and co-integrated as in equation (2), Engle and Granger (1987) show that the underlying data-generation process has an error-correction representation

$$A(B)\Delta x_t = m + \gamma z_{t-1} + \varepsilon_t \tag{3}$$

where B and  $\Delta$  are the lag and differencing operators, A(0) is the  $n \times n$  identity matrix, m a vector of constants and  $\gamma$  an  $n \times r$  matrix of coefficients. In equation (1),

$$\Delta x_{2t} = x_{1t} - x_{2,t-1} + \varepsilon_{2t}$$
  
=  $(x_{1,t-1} - x_{2,t-1}) + \varepsilon_{1t} + \varepsilon_{2t}$ 

so that

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} (x_{1,t-1} - x_{2,t-1}) + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{1t} + \varepsilon_{2t} \end{bmatrix}$$

Returning to the general case, if  $A(B) = I_n$ , i.e. if there are no lags of  $\Delta x_i$  in equation (3),

$$\Delta x_{1t} = m + \gamma_{11} z_{1,t-1} + \gamma_{12} z_{2,t-1} + \dots + \gamma_{1r} z_{r,t-1} + \varepsilon_{1t}$$
(4)

When a forecasted variable  $y_t$  is I(1), any reasonable forecast  $f_t$  should be co-integrated with it, with co-integrating vector  $(1, -1)^t$ . If not, the forecast error  $y_t - f_t$  will not be stationary and the series and its forecast will drift increasingly apart over time. Let  $\{f_{t,h}^i\}, i = 1, 2, ..., r$ 

be a set of r forecasts of  $y_{t+h}$  made at time t. If  $y_t - f_{t-h,h}^i$  and  $y_{t-h} - y_t$  are both I(0), then so is

$$z_{i,t-h} \equiv f_{t-h,h}^{i} - y_{t-h} \tag{5}$$

For h = 1, equations (4) and (5) suggest the relation

$$\Delta y_t = m + \beta_1 z_{1,t-1} + \dots + \beta_r z_{r,t-1} + \varepsilon_t \tag{6}$$

or

$$y_t = m + y_{t-1} + \beta_1 (f_{t-1,1}^1 - y_{t-1}) + \dots + \beta_r (f_{t-1,1}^r - y_{t-1}) + \varepsilon_t$$
(7)

Since  $f_{t-1,1}^{j} - y_{t-1}$  can be thought of as the *j*th forecaster's forecast of the change in  $y_t$ , equation (6) can be interpreted as explaining the change in y as a linear combination of forecasts of the change. This is not the same as a linear combination of forecasts of the level. When we rewrite equation (7) as

$$y_{t} = m + \left(1 - \sum_{i=1}^{r} \beta_{i}\right) y_{t-1} + \sum_{i=1}^{r} \beta_{i} f_{t-1,1}^{i} + \varepsilon_{t}$$
(8)

several differences become apparent. First, when combining forecasts of an integrated series, a lag of the forecasted series itself should be included in the combination. Second, the coefficients (excepting the constant m) in regression (8) should be constrained to add to unity. Third, if the coefficients  $\{\beta_i\}$  sum to unity the lagged dependent variable in regression (8) drops out and the remaining coefficients are constrained to add to unity. If the constant term is also dropped the Bates-Granger combination results. Fourth, since all the variables in equation (6) are I(0), the *t*-statistics of the regressors, including the constant *m*, can be used to decide whether to retain all the forecasts in the combination. Moreover, the  $\{z_{i,t}\}$  are likely to be less correlated with each other than are the original forecasts, so that the *t*-statistics on the coefficients are more accurate than is normally the case when combining forecasts via regression.

#### **ROBUST ESTIMATION**

Robust estimation, such as that delivered by minimizing absolute deviations, may be useful in several contexts. If the conditions of the Gauss–Markov theorem are satisfied for the population from which the sample is drawn but the sample exhibits gross errors, least squares can produce estimators with arbitrarily large biases, while a robust procedure by construction limits the influence of any observation. However, least squares is still asymptotically the best linear unbiased estimator, so robust estimation techniques in this context are interesting only for small samples. Another context is the case of contaminated data in which the number of gross errors in the data is a function of the number of observations, the classical example being keypunch or other non-zero mean errors. In this case the Gauss–Markov assumptions fail and least squares produces inconsistent estimates, even in large samples. Since exercises in combining forecasts are commonly performed on samples of less than 50 observations, robust estimation procedures may be more appropriate than least squares, even if we believe that gross errors in forecasts are not a function of the number of observations.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Non-quadratic cost functions can also lead to situations where particular robust estimators are optimal even asymptotically. For a fuller discussion of the appropriateness of robust procedures, see Hampel *et al.* (1986, pp. 21-31).

# RANKING FORECASTS

The use of the mean square error criterion to rank forecasts is an accepted procedure in the literature on combining forecasts. This criterion is widely used in statistical analysis and permits no ambiguity. Its disadvantages include the unambiguous ranking it assigns and the lack of tests of significant differences in mean square error when models are non-nested.

Ambiguity may be appropriate when comparing performance of combining algorithms on data out of the sample on which the combining weights are formed under the following conditions. Given two sets of combined forecasts (say, produced by least squares and least absolute deviations) and that the combinations incorporate the relevant information set in only a partly overlapping manner, neither is an optimal combined forecast for the information set and neither is unambiguously better than the other. In other words, both can forecast something the other cannot—neither 'encompasses' the other, though one will almost always have a smaller mean square error.

A procedure addressing these criticisms issues from the encompassing literature. Specifically, a formal test of 'encompassing in forecast' of two models, as suggested by Chong and Hendry (1986), is suitable to rank combined forecasts out-of-sample. If both combinations incorporate information in only partly overlapping ways neither is ranked above the other, and tests of significance are immediate. In what follows the alternative combining algorithms can be thought of as 'models' and the 'forecasts' as the combined forecasts we are interested in evaluating.

The encompassing literature provides a unifying framework for non-nested hypothesis testing. Chong and Hendry focus in part on evaluating alternative non-nested models for their relative forecasting ability. The idea of encompassing is to formalize the intuition that one model is better than another if it can predict what the other model will yield given a certain information set and the other model cannot do the same.

The encompassing test is a test of the significance of the coefficients from a standard least squares regression of the forecasts being compared on the variable being forecasted. To be precise, if the null is model 1 encompasses model 2 in forecast, then subtract, from the variable being forecasted, model 1's forecast, and regress this residual on the forecast from model 2. If model 2's forecast coefficient is insignificant at some predetermined level we do not reject the null that model 1's forecast encompasses the other. If it is found that model 2 does not encompass model 1 in forecast at the same level of significance (i.e. the coefficient of model 1's forecast is significant), model 1 is ranked above model 2. It should be reiterated that this paper is testing for encompassing on out-of-sample data.

Although seemingly incongruent, multicollinearity could lead to two forecasts apparently encompassing each other, since the null of the test is of encompassing and multicollinearity reduces the power of the test. In this case neither model would rank above the other, as neither forecast contains information the other does not. If no model encompasses another (both models' forecasts have significant coefficients in the appropriate regressions) then, again, neither ranks above the other. We expect *a priori* that the mean square error criterion will not contradict the ranking assigned by the encompassing criterion.

# DATA AND RELATED ISSUES

The dependent variable data series used in this study is the total unemployment rate as a percentage of the civilian labor force, seasonally adjusted, for persons 16 years and over, for

the USA from 1948:1 to 1986:1 on a monthly basis. Two models are constructed and forecasts from them obtained. Details of model construction and data sources may be found in the Appendix.

For both models the first fourteen observations are used only to extract the lags needed for the model specification. On the next 310 observations (up to 1975, approximately) the model parameters are estimated. Using these estimates, one-step-ahead forecasts for the next 120 observations are made. The models at this step could be re-estimated and parameter estimates updated with each observation, as a forecaster in real time would do. However, the object of this study is not to investigate alternative modeling performance over time. It is to determine the relative performance of combining algorithms for *given* forecasts. Hence forecasts produced by the models, dispensing with the unpdating procedure, were judged appropriate.

However, an updating procedure is employed in the formation of combining weights. Updating here is viewed as essential, as this is what an agent interested in combining would do with a given set of forecasts in real time. A rolling forecast window is employed, with 60 out-of-sample forecasts used to estimate weights to make the combined forecast for the sixty-first period. Thus the set of 120 out-of-sample forecasts yielded 60 out-of-sample one-step combined forecasts for each combining rule. It is these 60 combined forecasts for each combining rule that are analyzed below to determine performance. Naturally, different windows could be chosen (say, 40 observations) which would still leave enough degrees of freedom to estimate the model parameters with reasonable precision. Alternatively, a declining weight could be attached to earlier observations, avoiding the truncation of a rolling window. Bates and Granger discuss such schemes in some detail.

Some observations regarding the data and implications for estimation must be made here:

- (1) We know that robust procedures will outperform non-robust procedures if we have influential outliers. An analysis of the unemployment variable on the out-of-sample data points to the presence of one or two gross outliers at most. Thus we can expect less than significant improvements with the robust procedures.
- (2) If the data are integrated of order one the analysis outlined earlier in this paper applies. The data series used here appears to be integrated of order one, as the Dickey-Fuller test fails to reject the unit root hypothesis.

# EMPIRICAL RESULTS

We address the most obvious question first. Is combining appropriate at all? In the period over which the weights are formed, the root mean squared forecast error (RMSFE) of model I is more than twice that of model II, casting suspicion on the usefulness of combining. However, the Chong and Hendry test of encompassing suggests that combining of forecasts is warranted here. Using White's (1982) standard errors, we can reject the hypotheses of either model I or model II's forecast encompassing the other. Their forecasts have *t*-statistics of 3.4 and 2.2, respectively, in the encompasing test regressions described in the previous section. The test is performed on the last 60 observations of the 120 out-of-sample forecasts to correspond to the sample on which similar encompassing tests of the combined forecasts are performed. Model II incorporates an interest rate term and is greatly affected by the 1980 credit controls experience. This is the primary cause of its large root mean squared error.

Three combining algorithms are investigated. All are special cases of the weighted

combination

Table 1

$$y_t = \alpha + \beta_0 y_{t-1} + \beta_1 f_{t-1,1}^1 + \beta_2 f_{t-1,1}^2$$
(8)

The combining schemes are (1) static unconstrained least squares ( $\beta_0 = 0$ ), (2) Bates-Granger constrained least squares ( $\alpha = \beta_0 = 0$ , and  $\beta_1 + \beta_2 = 1$ ) and (3) least squares with the contraints implied by co-integration ( $\alpha = 0$ , and  $\beta_0 + \beta_1 + \beta_2 = 1$ ). All three combinations are estimated by ordinary least squares, trimmed least squares (trim set at 5%, 10% and 15%), Least absolute deviations, and by the Cauchy, Fair, Huber, logistic and Andrews robust estimation procedures. Thirty sets of combining weights in all are obtained. Since all the robust procedures produce similar results, only the 15% trimmed least squares with co-integration constraint is reported. This is the best of the robust methods by RMSFE.

For the procedure that was used to estimate the trimmed least squares estimators, see Ruppert and Carroll (1980) and their discussion of the Kroenker and Bassett estimator. A proper description of this is beyond the scope of this paper, but an intuitive understanding can be gained by constructing an asymptotically equivalent estimator. Start with a preliminary estimator (e.g. least squares) and find the residuals. Remove from the data those observations corresponding to the 15% largest negative and 15% largest positive residuals. Then re-estimate the model with least squares on the remaining data to obtain the 15% trimmed least squares estimates. The other robust estimation procedures are type M estimators (i.e. they are generalizations of the usual maximum likelihood estimators). Instead of maximizing the likelihood function they minimize an alternative function of the regression residuals. For instance, in the case of the Cauchy, the function is

$$c^2 \sum_{t} \ln \left[ 1 + \left( \frac{u_t}{c} \right)^2 \right]$$

where  $u_t$  represents the residual and c = 2.3849 provides 0.95 asymptotic efficiency on the standard normal distribution. The Fair makes use of

$$2c^2 \sum_{t} \left[ \frac{|u_t|}{c} - \ln\left(1 + \frac{|u_t|}{c}\right) \right]$$

with c = 1.3998. The Huber and Andrews methods both make use of cosines and linear functions of the residuals.<sup>2</sup>

Variable	Mean	Standard deviation	Minimum value	Maximum value
Unemployment	8.32	1.24	6.30	10.70
Forecast 1	8.38	1.26	6.11	10.88
Forecast 2	8.22	1.23	6.03	10.66
Average	8.30	1.25	6.07	10.77
Least squares	8.31	1.26	6.08	10.89
Constrained least squares	8.30	1.26	6.04	10.80
Least squares with co-integration restrictions Trimmed LS (15%) with co-integration	8.31	1.26	6.03	10.80
restrictions	8.29	1.27	6.02	10.79

 $^{2}$ For a detailed description of the algorithms performing these robust estimations see Coleman *et al.* (1980) and Ruppert and Carroll (1980).

Forecast	Root mean squared forecast error
Forecast 1	0.2158
Forecast 2	0.2147
Average	0.2106
Least squares	0.2215
Constrained least squares	0.2163
Least squares with co-integration restrictions Trimmed LS (15%) with co-integration	0.2200
restrictions	0.2130

Table 2

Table 1 gives summary statistics for the out-of-sample forecast obtained using various estimators and Table 2 reports the root mean square one-step-ahead forecast errors for the same estimators as Table 1.

Ranking by the RMSFE criterion, a simple average is preferable to all the in-sample tailored procedures promoted as likely candidates for out-of-sample optimal performance. Of all the procedures only the last performs better than the individual forecasts.

The results of Clemen and Holden and Peel are that *all* their combining methods are inferior by the RMSFE criterion to the best of the individual forecasts, sometimes to all the individual forecasts. This study does produce some combined forecasts<sup>3</sup> that outperform both individual forecasts in RMSFE.

Tests of encompassing on the out of sample data<sup>4</sup> show that all the combined forecasts encompass the individual forecasts and themselves have significant coefficients when the individual forecasts are tested to encompass them (i.e. they are not encompassed in turn by the individual forecasts). This is something arbitrary combinations of the individual forecasts are not guaranteed to do. Following the ranking by the encompassing procedure, all the combining methods are 'better' than the individual forecasts. Pairwise encompassing tests on the combined forecasts show each of them encompassing the others, probably due to multicollinearity.

## CONCLUSIONS

Commonly encountered with economic data are integrated series and contaminated data. This study demonstrates that improved forecasts may be produced by incorporating co-integration restrictions when combining integrated forecasts. Robust procedures performed no worse than least squares and promise to do better.

That procedures ranked above the individual forecasts by the encompassing procedure are simultaneously ranked below these by the RMSFE criterion argues most strongly against the appropriateness of using the RMSFE ranking procedure alone, particularly if the differences in RMSFE are very small. This suggests that the encompassing procedure may be profitably combined with the RMSFE criterion. It would be interesting to see if similar results obtained for recent studies in which individual forecasts performed 'better' than all combining efforts.

<sup>&</sup>lt;sup>3</sup>A number of the robust procedures not reported here also had smaller RMSFE than both the individual forecasts. <sup>4</sup>Within sample, an OLS combination trivially encompasses its components, since OLS residuals are orthogonal to independent variables.

# APPENDIX: THE DATA AND REGRESSION RESULTS ON MODEL ESTIMATION PERIOD

The first of the economic forecasts considered is generated by a model with roots in the literature on rational expectations. The information set used for this model includes the unanticipated inflation rate, lags to order four of the dependent variable (both suggested by Sargent in his classical macroeconomic model) and lags of two variables in addition to these. These variables are a measure of labor cost per unit output as a percentage of trend, seasonally adjusted, and the six-month commercial paper rate (model I). The second model is a simple autoregressive time series one having as an information set overlapping but not encompassing or encompassed data (model II). These two models are not suggested to be serious contributions to the literature investigating empirical regularities of the unemployment rate.

The data come from the Citibase data set. The unemployment rate variable (LHUR) is the seasonally adjusted monthly percentage of the civilian labor force (both sexes) unemployed. Its *i*th lag is denoted *Ui*. The series started on January 1948; the first 14 observations are lost to lag generation. The term proxying for unexpected inflation as suggested by Sargent's rational expectations model is the lagged CPI minus the lagged time series one-step-ahead forecast of the CPI (LCPI). Lagged unexpected inflation is used to avoid contemporaneous correlation problems. The CPI series used is the seasonally unadjusted monthly all items index, normalized to 100 at 1967 (*PZUNEW*). The labor cost proxy variable is an estimate of labor cost per unit output as percentage of trend (*PLMDT*) and its *i*th lag is denoted *LABi*. The interest rate variable is the six-month commercial paper rate (*FYCP*) and its *i*th lag is denoted *Ii*. The regression results for models I and II are given below.

6		Sum of	Mean		
Source	DF	squares	square	F value	
Model	11	396.7534	36.06849	853.812	
Error	298	12.58873	0.04224405		
C Total	309	409.3421			
Root MSE		0.2055336	R-square	0.9692	
Dep mean		4.843548	ADJ R-SQ	0.9681	
C.V.		4.24345			
		_			
		Parameter	Standard	T for $H_0$ :	
Variable	DF	estimate	error	Parameter = 0	
INTERCEP	1	0.05270638	0.6385565	0.083	
Т	1	-0.000498957	0.0003734387	-1.336	
U1	1	1.10262	0.05756616	19.154	
U2	1	0.1210577	0.08563636	1.414	
<i>U</i> 3	1	-0.167439	0.08579784	-1.952	
U4	1	-0.0850563	0.06074414	-1.400	
LCP1	1	-0.44812	0.9155175	-0.489	
<i>I</i> 8	1	0.02911755	0.01493687	1.949	
LAB1	1	-0.00359161	0.002082027	-1.725	
LAB2	1	-0.004216	0.002100568	-2.007	
LAB4	1	0.02862356	0.01685554	1.698	
LAB5	1	-0.0204248	0.01579693	- 1.293	

Model I

Analysis of variance

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Model II Analysis of variance							
Source	DF	Sum of squares	Mean square	F value			
Model Error C total	7 302 309	396.8858 12.45634 409.3421	56.69796 0.04124617	1374.624			
	MSE mean C.V.	0.2030915 4.843548 4.193032	<i>R</i> -square ADJ R-SQ	0.9696 0.9689			
Variable	DF	Parameter estimate	Standard error	$T$ for $H_0$ : Parameter = 0			
INTERCEP	1	0.1322973	0.06102578	2.168			
<i>U</i> 1	1	1.162249	0.03808071	30.521			
U3 U6	1	- 0.10379 - 0.0716825	0.05545539 0.04367154	-1.872 -1.641			
U10	1	-0.120747	0.06673275	-1.809			
U11	1	0.1846761	0.0838996	2.201			
<i>U</i> 12	1	-0.258553	0.08372705	-3.088			
<i>U</i> 13	1	0.1814123	0.05665458	3.202			

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